**Ebanks, Bruce R.**

**Fundamental equation of information revisited.** (English) Zbl 0842.39008

Aequationes Math. 51, No. 1-2, 86-99 (1996).

Let $J$ be the $k$-th Cartesian power of the open interval $]0,1[$. The fundamental equation of information measures of multiplicative type, depending upon $k$ probability distributions with nonzero probabilities, is $f(x) + M(1-x)f(y/(1-x)) = f(y) + M(1-y)f(x/(1-y))$ (whenever $x$, $y$ and also $x+y$ are in $J$), where $M : J \to \mathbb{R}$ is multiplicative. This equation has been completely solved by *C. T. Ng* and the reviewer [Linear Algebra Appl. 52-53, 1-30 (1983; Zbl 0517.39006)].

The author gives here an alternative proof for the case where $M$ is also additive (as is the case for the most applied information measures) which uses a result of *B. Jessen*, *J. Karpf* and *A. Thorup* [Math. Scand. 22, 257-265 (1968; Zbl 0183.04004)] just once (instead of twice) and is more similar to the proof for the nonadditive case in the 1983 paper. Finally, generalizations of (part of) the result to more general domains (ordered commutative rings with unit, in particular real-closed fields and positive cones in ordered fields) are offered.

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**MSC:**

39B22 Functional equations for real functions
94A17 Measures of information, entropy
39B52 Functional equations for functions with more general domains and/or ranges

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ordered commutative rings; fundamental equation of information measures; positive cones in ordered fields

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