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Dirichlet forms and symmetric Markov processes. (English) Zbl 0838.31001
de Gruyter Studies in Mathematics. 19. Berlin: Walter de Gruyter. viii, 392 p. (1994).

This book is mainly based on the first named author's book [Dirichlet forms and Markov processes (1980; Zbl 0422.31007)] and might be considered as a second edition. The principle structure in revised form has been taken over to this new book, but a lot of new material which had been developed in the last decade is included. This especially concerns the description of global properties and of the additive functional of a Hunt process using Dirichlet forms. The book certainly is a standard reference for the theory of Dirichlet forms.

In the first, analytic part of the book the authors give the basic definition of a Dirichlet form, introduce the notion of extended Dirichlet spaces, and give criteria for global properties of the form and the corresponding semigroup such as transience, recurrence, conservativeness. In the next chapter they develop the potential-theoretical framework based on the notion of the capacity given by a Dirichlet form. In particular important notions like quasi-continuity, quasi everywhere, smooth measures are introduced and the spectral synthesis in Dirichlet spaces is described. Next they illustrate the scope of Dirichlet forms by giving criteria for the closability of a Markovian form in concrete examples. The Beurling-Deny formula is established, i.e. the decomposition of a regular Dirichlet form into a strongly local, a nonlocal jump, and a killing part. Considerations about the class of extensions of a closable Markovian form, Markovian and not Markovian, including again concrete examples conclude the analytic part.

Next the authors consider the probabilistic counterpart and give the probabilistic interpretation of the potential theoretical notions in terms of the corresponding symmetric Hunt process. A detailed investigation of the additive functionals of a Hunt process using Dirichlet forms follows. First they prove the Revuz correspondence between the class of positive continuous additive functionals and the class of smooth measures of the Dirichlet form. For functions u in the (extended) Dirichlet space (and later on for functions that are only locally in the Dirichlet space) the Fukushima decomposition of the additive functional $\tilde{u}(x_t) - \tilde{u}(x_0)$ is established, where \tilde{u} is a quasi continuous modification of u , i.e. its decomposition into a martingale additive functional and a continuous additive functional of zero energy (but not necessarily of bounded variation). The martingale part again has an analytic interpretation, as the measure which is in Revuz correspondence with the bracket of the martingale coincides with the energy measure of u . Also the zero energy part is studied in detail. The results are applied to stochastic integration and forward and backward martingale decomposition. In the next chapter different transformations of a Hunt process using additive functionals such as killing and time transformation and the corresponding perturbed Dirichlet form are considered.

The last chapter is devoted to the construction of the symmetric Hunt process associated to a regular Dirichlet form. It is also shown how the method applies if the underlying state space is not locally compact, but only a Lusin space, a situation that often occurs in infinite dimensional analysis.

In the appendix a short introduction to subjects like Choquet capacity, Hunt processes and martingale additive functionals is given. Moreover a representation of Dirichlet forms on nonlocally compact spaces by regular Dirichlet forms is shown, which was used in the preceding chapter to reduce the problem of constructing a Hunt process in the Lusin space case.

Reviewer: [W.Hoh \(Bielefeld\)](#)

MSC:

- 31-02** Research exposition (monographs, survey articles) pertaining to potential theory
- 60-02** Research exposition (monographs, survey articles) pertaining to probability theory
- 60J45** Probabilistic potential theory
- 31C25** Dirichlet forms

Cited in **14** Reviews
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Keywords:

additive functional; Hunt process; Dirichlet forms; extended Dirichlet spaces; capacity; Beurling-Deny formula; Revuz correspondence; Fukushima decomposition; stochastic integration; martingale decomposition