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Quadratic algebras associated with the union of a quadric and a line in \mathbb{P}^3 . (English)

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J. Algebra 165, No. 1, 63-90 (1994).

Summary: The author defines a family of graded quadratic algebras A_σ (on 4 generators) depending on a fixed nonsingular quadric Q in \mathbb{P}^3 , a fixed line L in \mathbb{P}^3 and an automorphism $\sigma \in \text{Aut}(Q \cup L)$. This family contains $\mathcal{O}_q(M_2(\mathbb{C}))$, the coordinate ring of quantum 2×2 matrices. Many of the algebraic properties of A_σ are shown to be determined by the geometric properties of $\{Q \cup L, \sigma\}$. For instance, when $A_\sigma = \mathcal{O}_q(M_2(\mathbb{C}))$, then the quantum determinant is the unique (up to a scalar multiple) homogeneous element of degree 2 in $\mathcal{O}_q(M_2(\mathbb{C}))$ that vanishes on the graph in $\mathbb{P}^3 \times \mathbb{P}^3$ of $\sigma|_Q$ but not on the graph of $\sigma|_L$. Following results of *M. Artin*, *J. Tate*, and *M. Van den Bergh* [“The Grothendieck Festschrift”, Prog. Math. 86, 33-85 (1990; Zbl 0744.14024); and Invent. Math. 106, 335-388 (1991; Zbl 0763.14001)], we study point and line modules over the algebras A_σ , and find that their algebraic properties are consequences of the geometric data. In particular, the point modules are in one-to-one correspondence with the points of $Q \cup L$, and the line modules are in bijection with the lines in \mathbb{P}^3 that either lie on Q or meet L . In the case of $\mathcal{O}_q(M_2(\mathbb{C}))$, when q is not a root of unity, the quantum determinant annihilates all the line modules $M(l)$ corresponding to lines $l \subset Q$; the determinant generates the whole annihilator for such $l \subset Q$ if and only if $l \cap L = \emptyset$.

MSC:

- 16S35 Twisted and skew group rings, crossed products
- 14A22 Noncommutative algebraic geometry
- 17B37 Quantum groups (quantized enveloping algebras) and related deformations
- 14H37 Automorphisms of curves
- 16W50 Graded rings and modules (associative rings and algebras)
- 14M07 Low codimension problems in algebraic geometry

Cited in **2** Reviews
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Keywords:

graded quadratic algebras; nonsingular quadrics; coordinate rings of quantum 2×2 matrices; quantum determinants; point modules; line modules

Full Text: DOI