Donovan, George C.; **Miller, Arnold R.**; **Moreland, Timothy J.**

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Am. Math. Mon. 100, No. 1, 53-58 (1993).

In the solution of equations by numerical methods, a commonly used stopping criterion is

(1) $|x_{n+1} - x_n| < \varepsilon$, where x_n is the nth term of the sequence generated by the method, and $\varepsilon > 0$ is the tolerance. However in general, criterion (1) can fail: there exist sequences (e.g., the partial sums of the harmonic series) for which (1) is true but which nonetheless diverge.

We derive two functions, which exhibit this "false convergence" phenomenon. The first of these has no real root, but nevertheless generates a sequence under Newton's method for which (1) is satisfied for any ε, namely, $\{\sqrt{u_n}\}$ where $u_n \in \mathbb{R}$, $u_{n+1} = u_n + 1$, and $n = 0, 1, \ldots$ Although this sequence satisfies (1), it obviously does not converge. The second function, like the first one, appears to converge where there are no roots, but it has a real root, to which Newton's method will never converge.

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