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Convergence of the Cahn-Hilliard equation to the Hele-Shaw model. (English) Zbl 0828.35105
Arch. Ration. Mech. Anal. 128, No. 2, 165-205 (1994).

Summary: We prove that level surfaces of solutions to the Cahn-Hilliard equation tend to solutions of the Hele-Shaw problem under the assumption that classical solutions of the latter exist. The method is based on a new matched asymptotic expansion for solutions, a spectral analysis for linearized operators, and an estimate for the difference between the true solutions and certain approximate ones.

MSC:

35Q35 PDEs in connection with fluid mechanics
35R35 Free boundary problems for PDEs
76D99 Incompressible viscous fluids

Cited in **2** Reviews
Cited in **112** Documents

Keywords:

level surfaces of solutions; Cahn-Hilliard equation; Hele-Shaw problem; asymptotic expansion for solutions; spectral analysis

Full Text: [DOI](#)

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