

Caporaso, Lucia

A compactification of the universal Picard variety over the moduli space of stable curves.

(English) [Zbl 0827.14014](#)

J. Am. Math. Soc. 7, No. 3, 589-660 (1994).

The author constructs a compactification for the relative degree- d Picard variety associated to a family of (proper) stable curves. The problem arises because the Picard functor neither is proper nor separated when fibers of X/S are not smooth. For instance one can find families of invertible sheaves specializing to sheaves not locally free when the central fiber has nodes. The author uses a new functor to solve this problem. One crucial idea is that in this new functor such families specialize to invertible sheaves on the curve resulting from replacing the node (in the central fiber) by \mathbb{P}^1 . [The same idea was used by *D. Gieseker* for rank-2 vector bundles on nodal curves; cf. *J. Differ. Geom.* 19, 173-206 (1984; [Zbl 0557.14008](#)).]The proof relies on GIT (=“geometric invariant theory”) theory.

Reviewer: Jun Li (Los Angeles)

MSC:

14H10 Families, moduli of curves (algebraic)
14D20 Algebraic moduli problems, moduli of vector bundles
14C22 Picard groups

Cited in **12** Reviews
Cited in **46** Documents

Keywords:

moduli problem; semistable curves; geometric invariant theory; compactification; Picard variety

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