Bartels, S. G.; Pallaschke, D.

**Some remarks on the space of differences of sublinear functions.** (English) Zbl 0826.49011

Appl. Math. 22, No. 3, 419-426 (1994).

For a real Banach space $X$ (with topological dual $X'$) let $\mathcal{D}(X)$ denote the space of differences of real-valued sublinear functions. The authors show that if $X$ is separable, then there exists a countable family of seminorms such that $\mathcal{D}(X)$ becomes a Fréchet space. In particular, if $X=\mathbb{R}^n$, the construction yields a norm such that $\mathcal{D}(\mathbb{R}^n)$ becomes a Banach space.

The following problem is also considered. Let $f: U\to\mathbb{R}$, $U\subseteq X$ open, be directionally differentiable at $x_0\in U$, with $\frac{df}{dg}\big|_{x_0}$ denoting the directional derivative of $f$ at $x_0$ in the direction $g$. A sublinear functional $p: X\to\mathbb{R}$ is called an upper convex approximation (u.c.a.) of $f$ at $x_0$ if

$$p(g)\geq \frac{df}{dg}\Big|_{x_0}\qquad\text{for all}\quad g\in X,$$

and a family $\Phi_{f,x_0}$ of u.c.a. of $f$ at $x_0$ is called exhaustive if

$$\inf_{p\in\Phi_{f,x_0}} p(g)=\frac{df}{dg}\Big|_{x_0}\qquad\text{for all}\quad g\in X.$$

The authors show that the family of all $p: g\mapsto \kappa|g|-\langle l,g\rangle$, $g\in X$, $\kappa\in\mathbb{R}_+$, $l\in X'$, contains an exhausitve family of u.c.a. of $f$ at $x_0$ provided that $f$ is quasidifferentiable at $x_0$ and $X'$ is smooth.

Reviewer: W.Schirotzek (Dresden)

**MSC:**

49J52 Nonsmooth analysis
26A27 Nondifferentiability (nondifferentiable functions, points of nondifferentiability), discontinuous derivatives
90C30 Nonlinear programming



**Keywords:**

quasidifferentiable function; differences; real-valued sublinear functions; directional derivative; upper convex approximation

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