Summary: Recently Stolarsky proved that the inequality
\[
\int_0^1 g(x^{1/(a+b)}) \, dx \geq \int_0^1 g(x^{1/a}) \, dx \int_0^1 g(x^{1/b}) \, dx
\]
holds for every \(a, b > 0\) and every nonincreasing function on \([0,1]\) satisfying \(0 \leq g(u) \leq 1\). In this paper, we prove a weighted version of this inequality. Our proof is based on a generalized Chebyshev inequality. In particular, our result shows that the inequality (*) holds for every function \(g\) of bounded variation. We also generalize another inequality by Stolarsky concerning the \(\Gamma\)-function.

MSC:

26D15 Inequalities for sums, series and integrals
26A45 Functions of bounded variation, generalizations
33B15 Gamma, beta and polygamma functions

Keywords:

functions of bounded variation; gamma function; Stolarsky inequality; weighted version; Chebyshev inequality

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