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Hyper Markov laws in the statistical analysis of decomposable graphical models. (English)

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Summary: This paper introduces and investigates the notion of a hyper Markov law, which is a probability distribution over the set of probability measures on a multivariate space that (i) is concentrated on the set of Markov probabilities over some decomposable graph, and (ii) satisfies certain conditional independence restrictions related to that graph. A stronger version of this hyper Markov property is also studied.

Our analysis starts by reconsidering the properties of Markov probabilities, using an abstract approach which thereafter proves equally applicable to the hyper Markov case. Next, it is shown constructively that hyper Markov laws exist, that they appear as sampling distributions of maximum likelihood estimators in decomposable graphical models, and also that they form natural conjugate prior distributions for a Bayesian analysis of these models.

As examples we construct a range of specific hyper Markov laws, including the hyper multinomial, hyper Dirichlet and the hyper Wishart and inverse Wishart laws. These laws occur naturally in connection with the analysis of decomposable log-linear and covariance selection models.

MSC:

62H99 Multivariate analysis
05C90 Applications of graph theory
62E99 Statistical distribution theory
62E10 Characterization and structure theory of statistical distributions
62F15 Bayesian inference
60E99 Distribution theory

Keywords:

collapsibility; contingency tables; cut; Dirichlet distribution; expert systems; hyper Dirichlet law; hyper inverse Wishart law; hyper matrix F law; hyper matrix t law; hyper normal law; hyper multinomial law; hyper Wishart law; inverse Wishart distribution; log-linear models; matrix F distribution; matrix t distribution; triangulated graphs; Wishart distribution; meta Markov models; hyper Markov law; Markov probabilities; decomposable graph; conditional independence; sampling distributions of maximum likelihood estimators; graphical models; natural conjugate prior distributions; covariance selection models

Full Text: DOI