Colbourn, Charles J.; **Mendelsohn, Eric**; **Rosa, Alexander**; **Širáň, Jozef**
Anti-mitre Steiner triple systems. (English) Zbl 0815.05017
Graphs Comb. 10, No. 3, 215-224 (1994).

A (k,ℓ)-configuration in a Steiner triple system (V,B), is a subset of ℓ triples from B whose union is a k-element subset of V. The Pasch configuration is the $(6,4)$-configuration on a set $\{a,b,c,d,e,f\}$ with triples abe, acf, bdf, cde. The mitre is the $(7,5)$-configuration on a set $\{a,b,c,d,e,f,g\}$ with triples abe, acf, adg, bcd, efg. A Steiner triple system (STS) is anti- Pasch (anti-mitre) if it does not contain any Pasch (mitre) configuration. Moreover, an STS is called r-sparse if every set of $r+2$ elements carries fewer than r triples. Every STS is 3-sparse, is 4-sparse if and only if it is anti-Pasch, and 5-sparse if and only if it is both anti-Pasch and anti-mitre.

This paper makes substantial progress toward characterizing those v for which there exists an anti-mitre STS of order v, and shows that for at least $\frac{9}{16}$ of the admissible values of v there exists an anti-mitre STS. The paper includes a table summarising small cyclic STS up to order 57, with the number of cyclic STS which are anti-Pasch or anti-mitre, or both (5-sparse). Also cyclic 5-sparse STS(v) are given for orders $v=19$ and ($v\equiv 1$ or $3 \pmod 6$), $33\leq v\leq 97$. This leads to the conjecture made in the paper that a 5-sparse STS(v) exists for all $v\equiv 1,3 \pmod 6$, $v\geq 33$. Existence of 5-sparse STS for orders 21, 25, 27 and 31 remains open, while not a single example of a 6-sparse STS(v) is currently known.

Reviewer: E.J.Billington (Brisbane)

MSC:

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