Koenker, Roger; **Ng, Pin**; **Portnoy, Stephen**
Quantile smoothing splines. (English) Zbl 0810.62040
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Summary: Although nonparametric regression has traditionally focused on the estimation of conditional mean functions, nonparametric estimation of conditional quantile functions is often of substantial practical interest. We explore a class of quantile smoothing splines, defined as solutions to

$$\min_{g\in\mathcal{G}} \sum \rho_\tau\{y_i - g(x_i)\} + \lambda\left(\int_0^1 |g''(x)|^p dx\right)^{1/p},$$

with $\rho_\tau(u) = u\{\tau - I(u < 0)\}$, $p \geq 1$, and appropriately chosen $\mathcal{G}$. For the particular choices $p = 1$ and $p = \infty$ we characterise solutions $\widehat{g}$ as splines, and discuss computation by standard l_1-type linear programming techniques. At $\lambda = 0$, $\widehat{g}$ interpolates the τth quantiles at the distinct design points, and for λ sufficiently large $\widehat{g}$ is the linear regression quantile fit to the observations. Because the methods estimate conditional quantile functions they possess an inherent robustness to extreme observations in the y_i's. The entire path of solutions, in the quantile parameter τ, or the penalty parameter λ, may be efficiently computed by parametric linear programming methods. We note that the approach may be easily adapted to impose monotonicity and/or convexity constraints on the fitted function. An example is provided to illustrate the use of the proposed methods.

MSC:

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bandwidth selection; nonparametric regression; nonparametric estimation of conditional quantile functions; quantile smoothing splines; linear programming; linear regression quantile; conditional quantile functions; robustness to extreme observations

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