

**Gillet, Henri; Shalen, Peter B.; Skora, Richard K.**

**Simplicial approximation and low-rank trees.** (English) Zbl 0794.20038  
Comment. Math. Helv. 66, No. 4, 521-540 (1991).

Let  $\Lambda$  be a subgroup of  $\mathbb{R}$ . An action without inversion of a finitely generated group  $\Gamma$  on a  $\Lambda$ -tree  $T$  defines a (translation) length function  $\ell$  on  $\Gamma$  taking non-negative values in  $\Lambda$ . It is an open problem whether the given action can always be “simplicially approximated”, in the sense that there is a sequence  $(\ell_i)_{i \geq 0}$  of length functions defined by actions of  $\Gamma$  on  $\mathbb{Z}$ -trees, and a sequence  $(n_i)_{i \geq 0}$  of positive integers, such that  $\lim_{t \rightarrow \infty} \ell_i(\gamma)/n_i = \ell(\gamma)$ . In the language of the papers of *M. Culler* and *J. Morgan* [Proc. Lond. Math. Soc., III. Ser. 55, 571-604 (1987; [Zbl 0658.20021](#))] and *J. W. Morgan* and *P. B. Shalen* [Ann. Math., II. Ser. 120, 401-476 (1984; [Zbl 0583.57005](#))] this says that the projectivized length function defined by the given action is the closure of the set of projectivized length functions defined by simplicial actions. A second question arises in the case that the given action is small: can one take the approximating length functions  $\ell_i$  to be defined by small simplicial actions?

The main result of the reviewed paper gives affirmative answers to these questions when  $\Gamma$  is finitely presented and  $\Lambda$  has  $\mathbb{Q}$ -rank at most 2, assuming, in the rank-2 case, that the action satisfies the ascending chain condition. In particular, it implies that the second question has an affirmative answer if  $\Lambda$  has  $\mathbb{Q}$ -rank at most 2 and the small subgroups of  $\Lambda$  are finitely generated. The authors also observe that the results remain true if  $\Gamma$  is assumed to be finitely generated, rather than finitely presented, but the given action is assumed to be free.

Reviewer: [N.I.Osetinski \(Moskva\)](#)

**MSC:**

- [20E08](#) Groups acting on trees
- [05C05](#) Trees
- [57M15](#) Relations of low-dimensional topology with graph theory
- [20F65](#) Geometric group theory
- [05C25](#) Graphs and abstract algebra (groups, rings, fields, etc.)

Cited in **2** Documents

**Keywords:**

tree; translation length function; closure of projectivized length functions; finitely generated group; small simplicial actions; finitely presented

**Full Text:** [DOI](#) [EuDML](#)