Shih, Mau-Hsiang; **Lee, Shyh-Nan**

Combinatorial formulae for multiple set-valued labellings. (English) Zbl 0791.05007

Math. Ann. 296, No. 1, 35-61 (1993).

We develop the geometric notions of general position maps, π- balanced and π-subbalanced sets and then apply them to prove two general combinatorial formulae for multiple set-valued labellings on simplices related to the celebrated Sperner combinatorial lemma [Abh. Math. Semin. Univ. Hamb. 6, 265-272 (1928; JFM 54.0614.01)]. We apply one of the combinatorial formulae to covering theory of simplices and obtain a new covering theorem which is a common generalization of the Shapley theorem and the Gale theorem.

Reviewer: M.-H.Shih and S.-N.Lee (Chunq Li)

MSC:

05A99	Enumerative combinatorics	Cited in **2** Reviews
52B05	Combinatorial properties of polytopes and polyhedra (number of faces, shortest paths, etc.)	Cited in **2** Documents
57Q65	General position and transversality	
05D05	Extremal set theory	

Keywords:

position maps; multiple set-valued labellings; Sperner combinatorial lemma; covering theorem; Shapley theorem; Gale theorem

Full Text: DOI EuDML

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