Narang, T. D.; Garg, S. K.
On the uniqueness of best approximation in non-Archimedean spaces. (English)
Zbl 0789.46066

A. F. Monna [Indag. Math. 30, 484–496 (1968; Zbl 0172.39302)] has shown that
1. No subspace other than the trivial one of a non-Archimedean normed linear space (n.a. n.l.s.) $X$ over
a non-Archimedean non-trivially valued field $K$, can be Chebyshev and
2. Strict convexity in non-Archimedean normed linear spaces can not be defined analogously to that
available in normed linear spaces over the reals.

The authors first show by an example that a subset as against a subspace can be Chebyshev. They take
$X = Q_2$ (2-adic field) over itself and the subset to be $\{0, 1/2, 1/2^2, 1/2^3, 1/2^4, \ldots\}$. This subset is claimed
to be compact which is obviously wrong as it is not even bounded in $X$. The authors say that a n.a. n.l.s.
$X$ such that $\|X\| = |K|$, is strictly convex if $\|x + y\| = \max(\|x\|, \|y\|)$ and $\|x\| = \|y\| = 1$ imply $x = y$.
They have shown that if the characteristic of the residue class field of $K$ is 2 then no n.a.n.l.s. can be
strictly convex. Now let the characteristic be different from 2, and $\|X\| = |K|$. Choose $e, x \in X$ such
that $\|e\| = 1, \|x\| < 1$. Clearly $e + x \neq e - x$ while they meet the other requirements of the definition.
This means that no n.a.n.l.s. can be strictly convex in the sense of the definition given by the authors.

Reviewer: R. Bhaskaran (Madurai)

MSC:
46S10 Functional analysis over fields other than $\mathbb{R}$ or $\mathbb{C}$ or the quaternions; non-Archimedean func-
tional analysis
41A52 Uniqueness of best approximation

Keywords:
non-Archimedean normed linear space; non-Archimedean non-trivially valued field; Chebyshev; strictly
convex

Full Text: DOI

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Math.,30 (1968), 484–496.MR 40, 679. · Zbl 0172.39302
83j:46088. · Zbl 0499.46047
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