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Lattices in rank one Lie groups over local fields. (English) Zbl 0786.22017

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From the text: Let K be a local field, $\underline{G}$ a semi-simple algebraic K-group, $G = \underline{G}(K)$. In this paper we study lattices (i.e., discrete subgroups of finite covolume) in G when the K-rank of $\underline{G}$ is equal to one and K is a non-archimedean field. When K-rank$(\underline{G}) \geq 2$, Margulis (and Venkataramana) showed that every lattice is arithmetic. In contrast we have:

Theorem A. Assuming K is non-archimedean and K-rank$(\underline{G}) = 1$, then (i) G has a moduli space of cocompact lattices and, in particular, non-arithmetic ones. (ii) If char$(K) > 0$, the same holds also with non-uniform lattices.

Recall that when char$(K) = 0$ every lattice of G is cocompact. For rank one groups over $\mathbb{R}$ the question of existence of non- arithmetic lattices is meanwhile only still open for $SU(n, 1)$, $n \geq 4$. For a survey of known results the reader is referred to the author's paper [Bull. Am. Math. Soc. 20, 27-30 (1989; Zbl 0676.22007)] where the results of this paper were announced.

The proof of Theorem A gives lattices which are free products of hyperbolic cyclic groups and "cusp subgroups". Moreover we give a general structure theorem for lattices in G, from which Serre's conjecture that such arithmetic lattices do not satisfy the congruence subgroup property is confirmed.

MSC:

22E40 Discrete subgroups of Lie groups

20G25 Linear algebraic groups over local fields and their integers

Keywords:

cocompact non-arithmetic lattices; non-archimedean field; Serre's conjecture

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References:

BK H. Bass, R. Kulkarni, Uniform tree lattices, J. of the A.M.S. 3 (1990), 843–902. · Zbl 0734.05052

Be1 H. Behr, Finite presentability of arithmetic groups over global function fields, Proc. Edinburgh Math. Soc. 30 (1987) 23–39. · Zbl 0618.20033 · doi:10.1017/S0013091500017934

Be2 H. Behr, Nicht endlich erzeugte arithmetische gruppen ülner functionenkörpern, unpublished manuscript.

BH A. Borel, G. Harder, Existence of discrete cocompact subgroups of reductive groups over local fields, J. Reine Angew. Math. 298 (1978), 53–64. · Zbl 0385.14014

BoT1 A. Borel, J. Tits, Groupes reductifs, Publ. Math. I.H.E.S. 27 (1965), 55–150. · Zbl 0145.17402

BoT2 A. Borel, J. Tits, Éléments unipotents et sous-groupes paraboliques de groupes reductifs I, Invent. Math. 12 (1971), 95–104. · Zbl 0238.20055 · doi:10.1007/BF01404653

BT1 F. Bruhat, J. Tits, Groupes algébrique simples sur un corps local, Proc. Conf. on Local Fields (ed: T.A. Springer) (Driebergen) Springer-Verlag, New York, 1967, pp. 23–36.

BT2 F. Bruhat, J. Tits, Groupes réductifs sur un corps local I. Données radicielles valuées, Publ. Math. I.H.E.S. 41 (1972), 5–251.

Eb P. Eberlein, Lattices in spaces of non positive curvature, Ann. of Math. 111 (1980), 435–476. · Zbl 0432.53023 · doi:10.2307/1971104

Ef I. Efrat, On the existence of cusp forms over function fields, J. für die reine und angewandte Math. 399 (1989), 173–187. · Zbl 0667.10014 · doi:10.1515/crll.1989.399.173

GR H. Garland, M.S. Raghunathan, Fundamental domains for lattices inR-rank 1 semisimple Lie groups, Ann. of Math. 92 (1970), 279–326. · Zbl 0206.03603 · doi:10.2307/1970838

GP L. Gerritzen, M. van der Put, Schottky Groups and Mumford Curves, Springer Lecture Notes in Mathematics 817, Springer-Verlag, New York 1980. · Zbl 0442.14009

G M. Gromov, Hyperbolic groups, in "Essays in Group Theory" (Ed: S.M. Gersten) 75–264, MSRI publications No. 8, Springer-Verlag, New York 1987.

I Y. Ihara, On discrete subgroups of the two by two projective linear groups overp-adic fields, J. Math. Soc. Japan 18 (1966), 219–235. · Zbl 0158.27702 · doi:10.2969/jmsj/01830219

L1 A. Lubotzky, Free quotients and the congruence kernel ofSL 2, J. of Alg. 77 (1982), 411–418. · Zbl 0495.20021 · doi:10.1016/0021-

8693(82)90263-0

L2 A. Lubotzky, Trees and discrete subgroups of Lie groups over local fields, Bull. A.M.S. 20 (1989), 27–30. · Zbl 0676.22007 · doi:10.1090/S0273-0979-1989-15686-3

L3 A. Lubotzky, Lattices of minimal covolume inSL 2: A non Archimedean analogue of Siegel's Theorem \(\mu \geqslant \tfrac{\pi }{{21}}\) , J. of the A.M.S. 3 (1990), 961–975. · Zbl 0731.22009

LZ A. Lubotzky, R.J. Zimmer, Variants of Kazhdan's property for subgroups of semi-simple groups, Israel J. of Math. 66 (1989), 289–299. · Zbl 0706.22010 · doi:10.1007/BF02765899

R1 M.S. Raghunathan, Discrete Subgroups of Lie Groups, Springer Verlag, New York, 1972. · Zbl 0254.22005

R2 M.S. Raghunathan, Discrete subgroups of algebraic groups over, local fields of positive characteristics, Proc. Indian Acad. Sci. (Math. Sci.) 99 (1989), 127–146. · Zbl 0689.22004 · doi:10.1007/BF02837800

S1 J.P. Serre, Le problème des groupes de congruence pour SL2, Ann. of Math. 92 (1970), 489–527. · Zbl 0239.20063 · doi:10.2307/1970630

S2 J.P. Serre, Trees, Springer Verlag, New York, 1980.

T1 J. Tits, Sur le groupe des automorphismes d'un arbre, in "Essays in Topology and Related Toplics, Mémoires dédicés à Georges de Rham", Springer-Verlag (1970), 188–211.

T2 J. Tits, Unipotent elements and parabolic subgroups of reductive groups II, in "Algebraic Groups-Utrecht 1985", Springer Lecture Notes in Math. 1271 (1987), 265–284.

T3 J. Tits, Travaux de Margulis sur les sous-groupes discrete de groupes de Lie, in "Sém. Bourbaki 28e année, 1975/6, Exp. 482, Springer Lecture Notes in Math. 576 (1977).

V T.N. Venkataramana, On superrigidity and arithmeticity of lattices in semi-simple groups over local fields of arbitrary characteristic, Invent. Math. 92 (1988), 255–306. · Zbl 0649.22008 · doi:10.1007/BF01404454