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**Statistical inference procedures for bivariate Archimedean copulas.** (English) Zbl 0785.62032  
*J. Am. Stat. Assoc.* 88, No. 423, 1034-1043 (1993).

Summary: A bivariate distribution function  $H(x, y)$  with marginals  $F(x)$  and  $G(y)$  is said to be generated by an Archimedean copula if it can be expressed in the form

$$H(x, y) = \varphi^{-1}[\varphi\{F(x)\} + \varphi\{G(y)\}]$$

for some convex, decreasing function  $\varphi$  defined on  $(0, 1]$  in such a way that  $\varphi(1) = 0$ . Many well-known systems of bivariate distributions belong to this class, including those of Gumbel, Ali-Mikhail-Haq-Thélot, Clayton, Frank, and Hougaard. Frailty models also fall under that general prescription.

This article examines the problem of selecting an Archimedean copula providing a suitable representation of the dependence structure between two variates  $X$  and  $Y$  in the light of a random sample  $(X_1, Y_1), \dots, (X_n, Y_n)$ . The key to the estimation procedure is a one-dimensional empirical distribution function that can be constructed whether the uniform representation of  $X$  and  $Y$  is Archimedean or not, and independently of their marginals. This semiparametric estimator, based on a decomposition of Kendall's tau statistic, is seen to be  $\sqrt{n}$ -consistent, and an explicit formula for its asymptotic variance is provided. This leads to a strategy for selecting the parametric family of Archimedean copulas that provides the best possible fit to a given set of data. To illustrate these procedures, a uranium exploration data set is reanalyzed. Although the presentation is restricted to problems involving a random sample from a bivariate distribution, extensions to situations involving multivariate or censored data could be envisaged.

**MSC:**

**62G05** Nonparametric estimation  
**62G30** Order statistics; empirical distribution functions  
**62G07** Density estimation

Cited in **5** Reviews  
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**Keywords:**

empirical process; Frailty model;  $U$  statistic; Archimedean copula; bivariate distributions; dependence structure; empirical distribution; semiparametric estimator; decomposition of Kendall's tau statistic; asymptotic variance; uranium exploration data set

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