Deville, Robert; Godefroy, Gilles; Zizler, Václav
Smoothness and renormings in Banach spaces. (English) Zbl 0782.46019
Pitman Monographs and Surveys in Pure and Applied Mathematics. 64. Harlow: Longman Scientific & Technical. New York: John Wiley & Sons, Inc.. 376 p. (1993).

The book is devoted to investigation and applications of different convexity and smoothness properties of norms on Banach spaces. The considerable part of the book is devoted to equivalent norms on Banach spaces which enjoy special properties of convexity and smoothness. (Norms are called equivalent if they induce the same topology.)

The theory of equivalent norms is already presented in the books: *M. M. Day*, Normed Linear Spaces, Third edition (1973; Zbl 0268.46013) and *J. Diestel*, Geometry of Banach spaces, Lecture Notes Math. 485 (1975; Zbl 0307.46009).

The present book is an important contribution to the subject because

(1) It is based mainly on results appeared after the publication of the book of J. Diesel.

(2) The presentation of the theory of equivalent norms and their applications is more systematical than those in the books of Day and Diestel.

(3) Many results contained in the books of Day and Diestel are presented with alternative (usually shorter) proofs.

On reviewer's opinion the present book will be very useful for those who work in Banach space theory and nonlinear analysis.

Short description of the contents.

Chapter I is devoted to duality mapping. Let X be a Banach space, X^* its dual space, $x \in X$. Recall that the duality mapping is the set valued mapping defined by:

$$J(x) = \big\{f \in X^* : f(x) = \|x\|,\ \|f\| = 1\big\}.$$

After recalling classical facts about the duality mapping the authors present different variational principles, Jayne-Rogers theorem on selectors of duality mappings and characterization of Asplund spaces.

Chapter II contains an introduction to basic techniques used in renorming of Banach spaces by smooth and rotund norms. The chapter starts with results on duality between rotundity and smoothness. The proofs of basic results on locally uniformly rotund renormings are based on transfer method introduced by G. Godefroy. In order to prove existence results for norms which are simultaneously rotund and smooth (Asplund averaging) the authors use Baire category theorem for metric space of equivalent norms. This approach to Asplund averaging is due to M. Fabian, L. Zajicek and V. Zizler. The chapter also contains definitions and description of properties of norms which are uniformly rotund, weakly uniformly rotund, w^* uniformly rotund and uniformly rotund in every direction. The last sections contain results on renormings of classical Banach spaces and on extension of rotund or smooth equivalent norms from a subspace to the whole space.

Chapter III is devoted to rough and octahedral norms and their applications. Recall that a norm $\|\cdot\|$ on a Banach space X is called rough if for some $\varepsilon > 0$ and all $x \in X$,

$$\limsup_{\|h\|\to 0} \frac{\|x+h\| + \|x-h\| - 2\|x\|}{\|h\|} \geq \varepsilon.$$

A norm $\|\cdot\|$ is called octahedral if for every finite-dimensional subspace F of X and every $\eta > 0$, there exists $y \in X$, $\|y\| = 1$ such that for every $x \in F$ we have

$$\|x+y\| \geq (1-\eta)(\|x\| + 1).$$

It turns out that differential mappings on Banach spaces with rough norms have "harmonic" behaviour.

More precisely, let U be a bounded open subset of a Banach space with a rough norm, then any continuous mapping of the closure of U into arbitrary Banach space that is differentiable in U is uniquely determined by its values on the boundary of U.

Some special kinds of octahedrality of norms are used to give alternative proofs of results of R. Haydon, E. Odell and H. P. Rosenthal on characterization of Banach spaces containing isomorphic copies of ℓ_1 and to give a generalization of V. P. Khavin-M. C. Mooney result on weak sequential completeness of L_1/H_1.

Chapter IV is devoted to martingale techniques in rotundity and smoothness. Here the results of P. Enflo and G. Pisier on uniformly convex and uniformly smooth norms on superreflexive spaces and the results of S. Troyanski on martingale characterization of Banach spaces admitting locally uniformly rotund norms are presented (among others).

Chapter V is devoted to spaces for which there exist many times differentiable real valued functions with bounded nonempty support. The existence of such functions is connected with such properties of Banach spaces as superreflexivity, cotype, existence of isomorphic copies of c_0 and ℓ_p.

Chapters VI and VII are devoted to nonseparable spaces. Here the following topics are considered: projectional resolutions of identity, renormings of spaces of continuous functions. In particular, the recent results of R. Haydon on spaces of continuous functions on tree spaces are presented.

Chapter VII is devoted to Hamilton-Jacobi equations in infinite dimensions and to smooth approximations of continuous functions on a Banach space.

Each chapter is supplied with extensive bibliography, guide to the literature and list of open problems.

Reviewer: M.Ostrovskij (Khar'kov)

MSC:

46B20	Geometry and structure of normed linear spaces	Cited in **12** Reviews
46B03	Isomorphic theory (including renorming) of Banach spaces	Cited in **387** Documents

Keywords:

Asplund averaging; convexity; smoothness; norms on Banach spaces; equivalent norms; duality mapping; variational principles; Jayne-Rogers theorem on selectors; characterization of Asplund spaces; renorming of Banach spaces by smooth and rotund norms; rotundity; Baire category theorem; rough and octahedral norms; superreflexive spaces; martingale characterization of Banach spaces admitting locally uniformly rotund norms; cotype; isomorphic copies of c_0 and ℓ_p; nonseparable spaces; projectional resolutions of identity; Hamilton- Jacobi equations in infinite dimensions