Heinonen, Juha; **Kilpeläinen, Tero**; **Martio, Olli**

Nonlinear potential theory of degenerate elliptic equations. (English) Zbl 0780.31001

Oxford Mathematical Monographs. Oxford: Clarendon Press. v, 363 p. (1993).

The book gives an introduction to nonlinear potential theory. Most of the considerations are done for the quasilinear possibly degenerate elliptic equation

$$-\mathrm{div} A(x, \nabla u) = 0,$$

that should be regarded as a perturbation of the weighted p-Laplace equation

$$-\mathrm{div}\big(w(x)\, |\nabla u|^{p-2} \nabla u\big) = 0.$$

The book starts with a careful investigation of weighted L^p-Sobolev spaces, which are canonically related to solutions of the equation $-\mathrm{div} A(x, \nabla u) = 0$, and their interplay with the corresponding weighted variational capacity. After these preparations the authors study solutions and supersolutions of the equation $-\mathrm{div} A(x, \nabla u) = 0$ and the connection with variational integrals. Then typical potential theoretic notions are investigated in the nonlinear case such as superharmonic functions, balayage, Perron's method, polar sets, fine topology and harmonic measures. Especially the role of quasiconformal and quasiregular mappings in nonlinear potential theory is examined.

Finally the book contains a brief survey of an axiomatic approach to nonlinear potential theory.

Reviewer: W.Hoh (Erlangen)

MSC:

31-02 Research exposition (monographs, survey articles) pertaining to potential theory
31C45 Other generalizations (nonlinear potential theory, etc.)
31D05 Axiomatic potential theory
35J70 Degenerate elliptic equations
31B35 Connections of harmonic functions with differential equations in higher dimensions

Keywords:

weighted Sobolev space; quasiconformal mapping; quasilinear degenerate elliptic equation; introduction to nonlinear potential theory; supersolutions; variational integrals; superharmonic functions; balayage; Perron's method; polar sets; fine topology; harmonic measures; quasiregular mappings