

Mundici, Daniele

Ulam games, Łukasiewicz logic, and AF C^* -algebras. (English) Zbl 0780.03030

Ann. Soc. Math. Pol., Ser. IV, Fundam. Inf. 18, No. 2-4, 151-161 (1993).

Ulam asked what is the minimum number of yes-no questions necessary to find an unknown number in a certain finite search space, if up to k of the answers may be erroneous. Identifying lies with distortions, solutions to this problem provide optimal adaptive k -error correcting codes. The usual case of error correcting coding theory corresponds to the no feedback case, when all questions are asked at the beginning, before all answers. Answers in Ulam's game do not obey classical logic. For, (i) equal answers to the same repeated question are more informative than single answers, and (ii) opposite answers do not lead to contradiction. In fact, as proved by the present author, the underlying logic of Ulam's game is the many-valued calculus of Łukasiewicz, with the classical two-valued calculus corresponding to the case of no lies. Now, as also proved by the author, approximately finite-dimensional (AF) C^* -algebras can be interpreted as Lindenbaum algebras of this calculus – commutative AF C^* -algebras corresponding to countable Boolean algebras. In this paper we discuss the relationship between game-theoretic notions, noncommutative logic notions, and their C^* -algebraic counterparts. For instance, continuous trace AF C^* -algebras correspond to generalized Ulam games with separable Boolean search space in which questions are clopen subspaces. From the logic-algebraic viewpoint, these games correspond to finite products of countable Post algebras. It is also shown that Grothendieck's functor K_0 gives a one-one correspondence between continuous trace AF C^* -algebras and countable lattice-ordered Specker groups with strong unit. For further information we refer the reader to the paper and to its bibliography. For very recent developments, the following joint papers of the author should be also mentioned: with *G. A. Elliott*, "A characterization of lattice-ordered Abelian groups" [*Math. Z.* 213, 179- 185 (1993)], with *R. Cignoli* and *G. A. Elliott*, "Reconstructing C^* -algebras from their Murray-von Neumann orders" [*Adv. Math.* 101, 166-179 (1993)], with *G. Panti*, "Extending addition in Elliott's local semigroup" [*J. Funct. Anal.* 117, 461-472 (1993)].

Reviewer: [D.Mundici \(Milano\)](#)

MSC:

- 03G25 Other algebras related to logic
- 46L80 K -theory and operator algebras (including cyclic theory)
- 94B99 Theory of error-correcting codes and error-detecting codes
- 03B50 Many-valued logic
- 06F15 Ordered groups
- 46L89 Other "noncommutative" mathematics based on C^* -algebra theory

Cited in **3** Reviews
Cited in **13** Documents

Keywords:

lattice-ordered groups; MV-algebras; error-correcting coding with feedback; continuous trace C^* -algebras; commutative AF C^* -algebras; noncommutative logic; generalized Ulam games; Post algebras; lattice-ordered Specker groups