Reddy, Satish C.; **Schmid, Peter J.**; **Henningson, Dan S.**
Pseudospectra of the Orr-Sommerfeld operator. (English) Zbl 0778.34060
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Given $\varepsilon > 0$, a complex number z is in the ε- pseudospectrum of a matrix A, or a closed linear operator A densely defined in a Hilbert space H, if z is the resolvent set of A and $\|(zI - A)^{-1}\| \geq \varepsilon^{-1}$ or z is in the spectrum of A; if A is a square matrix, z is equivalently an eigenvalue of $A+E$ for some perturbation matrix with $\|E\| \leq \varepsilon$. The paper deals with the pseudospectra and numerical range of the Orr- Sommerfeld $(0-S)$ operator S for plane Poiseuille flow. These sets are estimated numerically for discrete analogues of S and their sensitivity to perturbations, and the effect of changing the Reynold's number, investigated. In the authors' opinion the properties exhibited for the discrete $0 - S$ operator reflect those of S. To understand the observed behaviour of the pseudospectra, a model operator is considered. This is generated by the operator associated with the Airy equation and the boundary conditions $\varphi(\pm 1) = 0$ in $L^2(-1, 1)$ and has eigenvalues which are highly sensitive to perturbations. This sensitivity is shown to be related to the existence of solutions of the eigenvalue equation which satisfy the boundary conditions to within an exponentially small factor. Theoretical and practical implications of the results are discussed.

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MSC:

34L05 General spectral theory of ordinary differential operators
76E05 Parallel shear flows in hydrodynamic stability

Keywords:

pseudospectra; Orr-Sommerfeld operator; plane Poiseuille flow; sensitivity to perturbations; Airy equation; eigenvalues

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