Trabucho, L.; **Viaño, J. M.**

A new approach of Timoshenko's beam theory by asymptotic expansion method. (English)

Zbl 0777.73028
RAIRO, Modélisation Math. Anal. Numér. 24, No. 5, 651-680 (1990).

A generalization of Timoshenko's beam theory is obtained by applying the asymptotic expansion method to a mixed variational formulation of the three-dimensional linearized elasticity model. The Timoshenko's constants are defined in a clear way, and their dependence on the geometry of the cross-section and on Poisson's ratio is shown. Several numerical examples are given to show the relationship between the classical and new constants for different geometries.

Reviewer: Jiang Furu (Shanghai)

MSC:

74K10 Rods (beams, columns, shafts, arches, rings, etc.)

Keywords:

perturbation method; mixed variational formulation; three-dimensional linearized elasticity; Timoshenko's constants

Full Text: DOI EuDML

References:

[1] I. AGANOVIC, Z. TUTEK, A justification of the one-dimensional model of elastic beam. Math. Methods in Applied Sci., 8, 1986, pp. 1-14. Zbl0603.73056 MR870989 · Zbl 0603.73056 · doi:10.1002/mma.1670080133

[2] A. BERMUDEZ, J. M. VIAÑO, Une justification des équations de la thermoélasticite des poutres à section variable par des méthodes asymptotiques, RAIRO, Analyse Numérique, 18, 1984, pp. 347-376. Zbl0572.73053 MR761673 · Zbl 0572.73053

[3] F. BREZZI, On the existence uniqueness and approximation of saddle point problems arising from Lagrange multipliers. RAIRO, Analyse Numérique, Sér Rouge, 2, 1974, pp. 129-151. Zbl0338.90047 MR365287 · Zbl 0338.90047

[4] D. CAILLERIE, The effect of a thin inclusion of high rigidity in a elastic body, Math. Methods in Applied Sci., 2, 1980, pp 251-270. Zbl0446.73014 MR581205 · Zbl 0446.73014 · doi:10.1002/mma.1670020302

[5] P. G. CIARLET, A justification of the von Karman equations. Arch. Rat. Mcch. Anal., 73, 1980, pp 349-389 Zbl0443.73034 MR569597 · Zbl 0443.73034 · doi:10.1007/BF00247674

[6] P. G. CIARLET, \texttt{<\hskip-.5e<} Recent progress in the two-dimensional approximation of three-dimensional plate models in nonlinear elasticity\texttt{>\hskip-.5e>} In Numerical Approximation of Partial Differential Equations E. L. Ortiz, Editor, North-Holland, Amsterdam, 1987, pp 3-19. Zbl0612.73060 MR899776 · Zbl 0612.73060

[7] P. G. CIARLET, P. DESTUYNDER, A justification of the two dimensional linear plate model. J. Mécanique, 18, 1979, pp 315-344. Zbl0415.73072 MR533827 · Zbl 0415.73072

[8] P. G. CIARLET, P. DESTUYNDER, A justification of a nonlinear model in place theory. Comp. Methods Appl. Mech. Engrg. 17/18, 1979, pp. 227-258. Zbl0405.73050 MR533827 · Zbl 0405.73050 · doi:10.1016/0045-7825(79)90089-6

[9] A. CIMETIÈRE, G. GEYMONAT, H. LEDRET, A. RAOULT, Z. TUTEK, Une dérivation d'un modèle non lineaire de poutres à partir de l'élasticite tridimensionelle. C. R. A. S., tome 302, Sér. I, n^\circ 19, 1986, pp. 697-700. Zbl0593.73046 MR847757 · Zbl 0593.73046

[10] P. DESTUYNDER, Sur une justification des modèles de plaques et de coques par les méthodes asymptotiques. Thèse, Univ. P. et M. Curie, Paris, 1980.

[11] P. DESTUYNDER, Une théorie asymptotique de plaque minces en élasticité lineaire. Masson, Paris, 1986. Zbl0627.73064 MR830660 · Zbl 0627.73064

[12] G. DUVAUT, J. L. LIONS, Inequalities in Mechanics and Physics. Springer-Verlag, Berlin, 1976. Zbl0331.35002 MR521262 · Zbl 0331.35002

[13] C. DYM, I. SHAMES, Solid Mechanics, A variational approch. McGraw-Hill, New York, 1973. · Zbl 1279.74001

[14] B. M. FRAEJIS DE VEUBEKE, A course in elasticity Applied Mathematical Sciences, Vol. 29, Springer-Verlag, Berlin, 1979. Zbl0419.73001 MR533738 · Zbl 0419.73001

[15] Y. C. FUNG, Foundations of Solid Mechanics Prentice-Hall, Englewood Cliffs, N. J., 1965.

[16] I. HLAVACEK, J. NECAS, Mathematical Theory of Elastic and Elasto-Plastic Bodies; An Introduction Elsevier, New York, 1981. Zbl0448.73009 MR600655 · Zbl 0448.73009

[17] J. L. LIONS, Perturbations singulières dans les problèmes aux limites et en contrôle optimal Lect. Notes in Math., 323, Springer-Verlag, Berlin, 1973. Zbl0268.49001 MR600331 · Zbl 0268.49001

[18] R. D. MINDLIN, Influence of rotary inertia and shear on flexural motions of isotropic elastic plates. J. Appl. Mech., 18, 1951, pp. 31 38. Zbl0044.40101 · Zbl 0044.40101

[19] A. RAOULT, Construction d'un modèle d'évolution de plaques avec terme d'inertie de rotation. Annali di Mathematica Pura ad Applicata, 139, 1985, pp. 361-400. Zbl0596.73033 MR798182 · Zbl 0596.73033 · doi:10.1007/BF01766863

[20] A. RIGOLOT, Sur une théorie asymptotique des poutres. Thèse, Univ. P. et M. Curie, Paris, 1976.

[21] A. RIGOLOT, Sur une théorie asymtotique des poutres. J. Mécanique, 11, 1972, pp. 673-703. Zbl0257.73013 MR368552 · Zbl 0257.73013

[22] A. RIGOLOT, Sur la déformation due à l'effort tranchant dans les poutres droites élastiques. Ann. Inst. Techn. Bât. Trav. Publ., n^\circ 363, 1978, pp 34-52.

[23] S. P. TIMOSHENKO, On the correction for shear of the differential equation for transverse vibration of prismatic bars. Phil Mag, Ser 6, 41, 1921, pp 744-746.

[24] L. TRABUCHO, J. M. VIANO, \texttt{<\hskip-.5e<} Derivation of generalized models for linear elastic beams by asymptotic expansion methods \texttt{>\hskip-.5e>} In Applications of Multiple Scaling to Mechanics (P. G. Ciarlet and E. Sanchez-Palencia, Editors), Masson, Paris, 1987, pp 302-315. Zbl0646.73024 MR902000 · Zbl 0646.73024

[25] L. TRABUCHO, J. M. VIANO, Dérivation de modèles généralisés de poutres en élasticité par méthode asymptotique. C. R. Acad. Sc. Paris, tome 304, Ser I, n^\circ 11, 1987. Zbl0627.73015 MR886729 · Zbl 0627.73015

[26] L. TRABUCHO, J. M. VIANO, Existence and characterization of higher order terms in an asymptotic expansion method for linearized elastic beams. J. Asymptotic Analysis, 2, 1989, pp. 223-255. Zbl0850.73126 MR1020349 · Zbl 0850.73126

[27] L. TRABUCHO, J. M. VIAÑO, A dérivation of generalized Saint Venat's torsion theory from three dimensional elasticity by asymptotic expansion methods. J. Applicable Analysis, 31, 1988, pp 129-148. Zbl0637.73003 MR1017507 · Zbl 0637.73003 · doi:10.1080/00036818808839820

[28] L. TRABUCHO, J. M. VIAÑO, A justification of Vlasov s bending-torsion theory in elastic beams (to appear). · Zbl 0850.73126

[29] L. TRABUCHO, J. M. VIAÑO, Critical cross sections in Timoshenko's beam theory (to appear). · Zbl 0777.73028

[30] J. M. VIAÑO, Contribution à l'étude des modèles bidimensionels en thermoélasticité de plaques d'épaisseur non constante. Thèse, Univ. P. et M. Curie, Paris, 1983.

[31] J. M. VIAÑO, \texttt{<\hskip-.5e<} Generalizacion y justificacion de modelos unilaterales en vigas elàsticas sobre fundacion \texttt{>\hskip-.5e>} In. Actas del VIII C. E. D. Y. A., Santander, España, 1985.