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**Nonparametric estimation of hazard functions and their derivatives under truncation model.**

(English) [\[Zbl 0777.62039\]](#)

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Summary: Nonparametric kernel estimators for hazard functions and their derivatives are considered under the random left truncation model. The estimator is of the form of sum of identically distributed but dependent random variables. Exact and asymptotic expressions for the biases and variances of the estimators are derived. Mean square consistency and local asymptotic normality of the estimators are established. Adaptive local bandwidths are obtained by estimating the optimal bandwidths consistently.

**MSC:**

62G07 Density estimation

62G20 Asymptotic properties of nonparametric inference

62E20 Asymptotic distribution theory in statistics

**Keywords:**

adaptive bandwidth choice; Hájek projection; mean square error; tightness; exact and asymptotic expressions; mean square consistency; adaptive local bandwidths; kernel estimators; hazard functions; derivatives; random left truncation model; identically distributed dependent random variables; biases; variances; local asymptotic normality; optimal bandwidths

**Full Text:** [DOI](#)

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