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A three-step finite element method for unsteady incompressible flows. (English)

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Summary: This paper describes a three-step finite element method and its applications to unsteady incompressible fluid flows. The stability analysis of the one-dimensional purely convection equation shows that this method has third-order accuracy and an extended numerical stability domain in comparison with the Lax-Wendroff finite element method. The method is cost effective for incompressible flows, because it permits less frequent updates of the pressure field with good accuracy. In contrast with the Taylor-Galerkin method, the present three-step finite element method does not contain any new higher-order derivatives, and is suitable for solving nonlinear multi-dimensional problems and flows with complicated outlet boundary conditions. The three-step finite element method has been used to simulate unsteady incompressible flows, such as the vortex pairing in mixing layer. The properties of the flow fields are displayed by the marker and cell technique. The obtained numerical results are in good agreement with the literature.

MSC:

76M10 Finite element methods applied to problems in fluid mechanics
76D05 Navier-Stokes equations for incompressible viscous fluids

Keywords:

stability analysis; third-order accuracy; nonlinear multi-dimensional problems; vortex pairing; marker and cell technique

Full Text: DOI

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