Wooley, Trevor D.

Large improvements in Waring's problem. (English) Zbl 0754.11026

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This paper is a substantial improvement on Vaughan's "New Iterative Method" in Waring's problem which gave rise to remarkable improvements in this circle of ideas already, but the methods explained here essentially halve the number of variables required to solve Waring's problem. To be more precise, let $G(k)$ be the smallest number s such that all large natural numbers are the sum of s k-th powers of natural numbers. A classical estimate of Vinogradov dating from the 1960's states that $G(k) \leq 2k \log k + o(k \log k)$. Since then this has been improved only in the lower order terms, but in this paper it is shown that

$$G(k) \leq k \log k + k \log \log k + O(1).$$

The method becomes effective already when $k \geq 6$, in particular the new bound $G(6) \leq 27$ is proved along the way.

The new idea is explained in detail in §2 of the paper under review. Very roughly speaking, Vaughan's new iterative method depends on a procedure which preserves homogeneity in the more classical p-adic iterative method of Davenport. The price one has to pay is that only "first differences" of the relevant exponential sums can be taken effectively, in a sense. But now the author manages to handle higher differences nearly as effective as the first. This is where the improvement comes from. Crucial reading for the "additive" number theorist.

Reviewer: J.Brüdern (Göttingen)

MSC:

11P05 Waring's problem and variants
11P55 Applications of the Hardy-Littlewood method
11L15 Weyl sums

Keywords:

upper bounds for $G(k)$; representations of integers; sum of higher powers; Waring's problem; Vaughan's new iterative method; higher differences

Full Text: DOI