Taguchi, Dai; **Tanaka, Akihiro**; **Yuasa, Tomooki**

L^q-error estimates for approximation of irregular functionals of random vectors. (English)

Zbl 1481.65025

IMA J. Numer. Anal. 42, No. 1, 840-873 (2022).

Summary: In [*R. Avikainen*, Finance Stoch. 13, No. 3, 381–401 (2009; Zbl 1199.60198)] the author showed that, for any $p, q \in [1, \infty)$, and any function f of bounded variation in $\mathbb{R}$, it holds that $\mathbb{E}[|f(X)-f(\widehat{X})|^q] \leq C(p,q)\mathbb{E}[|X-\widehat{X}|^p]^{\frac{1}{p+1}}$, where X is a one-dimensional random variable with a bounded density, and $\widehat{X}$ is an arbitrary random variable. In this article we will provide multi-dimensional versions of this estimate for functions of bounded variation in $\mathbb{R}^d$, Orlicz-Sobolev spaces, Sobolev spaces with variable exponents and fractional Sobolev spaces. The main idea of our arguments is to use the Hardy-Littlewood maximal estimates and pointwise characterizations of these function spaces. We apply our main results to analyze the numerical approximation for some irregular functionals of the solution of stochastic differential equations.

MSC:

65C30 Numerical solutions to stochastic differential and integral equations
65C05 Monte Carlo methods
34K50 Stochastic functional-differential equations

Keywords:

Avikainen's estimates; functions of bounded variation; Orlicz-Sobolev spaces; Sobolev spaces with variable exponents; fractional Sobolev spaces; Hardy-Littlewood maximal estimates; stochastic differential equations; Euler-Maruyama scheme; multilevel Monte Carlo method

Full Text: DOI arXiv