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Transonic steady-states of Euler-Poisson equations for semiconductor models with sonic boundary. (English) Zbl 07465239

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In this paper the authors consider radial transonic solutions for the steady hydrodynamic model of semiconductors represented by Euler-Poisson (E-P) equations with sonic boundary in n-dimensions. It is considered a system of three partial differential equations (PDE) written in the form:

$\operatorname{div}(\rho\mathbf{u}) = 0$, $(\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla P/\rho = \nabla\Phi - \mathbf{u}/\tau$, $\triangle\Phi = \rho - b(x)$, where $x \in \mathbb{R}^n$ $(n = 2, 3)$, $\rho(x)$ is the density of electrons, $\mathbf{u}$ presents the average particle velocity at location x, and $\Phi(x)$ denotes the electrostatic potential of electrons. The pressure $P = P(\rho)$ depends on the density for the isothermal flow with the constant temperature $T > 0$. The constant $\tau > 0$ represents the momentum relaxation time, and the known function $b(x) > 0$ is the doping profile standing for a background density of changed ions.

The authors investigate the structure of radial transonic solutions to E-P equations in an annulus domain which is defined by: $\mathcal{A} = \{x \in \mathbb{R}^n | r_0 < |x| < r_1\}$ with fixed constants $r_0 \in (0, r_1)$, where the inner boundary is given by $\Gamma_0 = \{x \in \mathbb{R}^n : |x| = r_0\}$, and the outer boundary $\Gamma_1 = \{x \in \mathbb{R}^n : |x| = r_1\}$. The closure of $\mathcal{A}$ is $\overline{\mathcal{A}} = \Gamma_0 \cup \mathcal{A} \cup \Gamma_1$. The goal of this paper is to show the existence results about radial transonic solutions to the E-P system with sonic boundary. It is given a sonic density ρ_0 and prescribed constant current j_0 at the inner boundary Γ_0, and a sonic density ρ_1 at the outer boundary Γ_1. It is shown that there exist infinitely many radial transonic shock steady-states of E-P system with a large relaxation time and infinitely many radial differentiable transonic steady-states of E-P system with a small relaxation time. It is investigated a two-dimensional system. It is shown that there exists a transonic shock solution over $[r_0, r_1]$ of this system satisfying both the entropy condition and Rankine-Hugoniot condition at a point $x_0 \in (r_0, r_1)$. Because of the arbitrary choices of x_0, the transonic shock solutions are infinitely many.

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MSC:

35Q81	PDEs in connection with semiconductor devices
35G50	Systems of nonlinear higher-order PDEs
35B40	Asymptotic behavior of solutions to PDEs
35B65	Smoothness and regularity of solutions to PDEs
35C06	Self-similar solutions to PDEs
35L67	Shocks and singularities for hyperbolic equations
76H05	Transonic flows
76L05	Shock waves and blast waves in fluid mechanics
76J20	Supersonic flows
78A35	Motion of charged particles
35A01	Existence problems for PDEs: global existence, local existence, non-existence
35Q35	PDEs in connection with fluid mechanics
35Q60	PDEs in connection with optics and electromagnetic theory

Keywords:

hydrodynamic model of semiconductors; Euler-Poisson equations; transonic shock solutions; smooth transonic solutions; local analysis

Full Text: DOI

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