

**Mahmudov, Elimhan N.**

**Optimization of higher-order differential inclusions with special boundary value conditions.**  
(English) [Zbl 1481.49022](#)

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Summary: The paper is devoted to Lagrange problem of optimal control theory with higher-order differential inclusions (HODI) and special boundary conditions. Optimality conditions are derived for HODIs, as well as for their discrete analogy. In this case, discretization method of the second-order differential inclusion is used to form sufficient optimality conditions for HODIs and periodic boundary conditions, the so-called transversality conditions. And to construct an Euler-Lagrange-type inclusion, a locally adjoint mapping is used, which is closely related to the coderivative concept of Mordukhovich. In turn, this approach requires several important equivalence results concerning LAMs to the discrete and discrete-approximate problems. The results obtained are demonstrated by the optimization of some “linear” optimal control problems, for which the Weierstrass-Pontryagin maximum principle and transversality conditions are formulated.

**MSC:**

[49K15](#) Optimality conditions for problems involving ordinary differential equations

[90C26](#) Nonconvex programming, global optimization

[93C15](#) Control/observation systems governed by ordinary differential equations

**Keywords:**

discrete-approximate; Euler-Lagrange; Hamiltonian; differential inclusion; equivalence

**Full Text:** [DOI](#)

**References:**

- [1] Alber, YI; Burachik, RS; Iusem, AN, A proximal point method for nonsmooth convex optimization problems in Banach spaces, *Abstract Appl. Anal.*, 2, 97-120 (1997) · [Zbl 0947.90091](#) · [doi:10.1155/S1085337597000298](#)
- [2] Arthi, G.; Balachandran, K., Controllability of damped second-order impulsive neutral functional differential systems with infinite delay, *J. Optim. Theory Appl.*, 152, 799-813 (2012) · [Zbl 1237.93023](#) · [doi:10.1007/s10957-011-9926-z](#)
- [3] Auslender, A.; Mechler, J., Second order viability problems for differential inclusions, *J. Math. Anal. Appl.*, 181, 205-218 (1994) · [Zbl 0808.34012](#) · [doi:10.1006/jmaa.1994.1015](#)
- [4] Azzam, DL; Castaing, C.; Thibault, L., Three boundary value problems for second order differential inclusions in Banach spaces, *Contr. Cyber.*, 31, 3, 659-693 (2002) · [Zbl 1111.34303](#)
- [5] Azzam, DL; Makhlof, A.; Thibault, L., Existence and relaxation theorem for a second order differential inclusion, *Numer. Funct. Anal. Optim.*, 31, 1103-1119 (2010) · [Zbl 1223.34018](#) · [doi:10.1080/01630563.2010.510982](#)
- [6] Benchohra, M.; Graef, JR; Henderson, J.; Ntouyas, SK, Nonresonance impulsive higher order functional nonconvex-valued differential inclusions, *Electr. J. Qual. Theory Differ. Equ.*, 13, 1-13 (2002) · [Zbl 1025.34084](#)
- [7] Benchohra, M.; Ntouyas, SK, Controllability for an infinite-time horizon of second-order differential inclusions in Banach spaces with nonlocal conditions, *J. Optim. Theory Appl.*, 109, 85-98 (2001) · [Zbl 0979.93008](#) · [doi:10.1023/A:1017561821201](#)
- [8] Cannarsa, P.; Frankowska, H.; Scarinci, T., Sensitivity relations for the Mayer problem with differential inclusions, *ESAIM: COCV*, 21, 789-814 (2015) · [Zbl 1319.49036](#)
- [9] Cernea, A., On the existence of solutions for a higher order differential inclusion without convexity, *Electr. J. Qual. Theory Differ. Equ.*, 8, 1-8 (2007) · [Zbl 1123.34046](#)
- [10] Chalishajar, DN, Controllability of second order Impulsive neutral functional differential inclusions with infinite delay, *J. Optim. Theory Appl.*, 154, 672-684 (2012) · [Zbl 1260.49023](#) · [doi:10.1007/s10957-012-0025-6](#)
- [11] Chang, YK; Li, WT, Controllability of second-order differential and integro differential Inclusions in Banach spaces, *J. Optim. Theory Appl.*, 129, 77-87 (2006) · [Zbl 1136.93005](#) · [doi:10.1007/s10957-006-9044-5](#)
- [12] Cibulka, R.; Dontchev, AL; Veliov, VM, Graves-type theorems for the sum of a Lipschitz function and a set-valued mapping, *SIAM J. Control Optim.*, 54, 6, 3273-3296 (2016) · [Zbl 1372.47070](#) · [doi:10.1137/16M1063150](#)
- [13] Conway, BA; Larson, KM, Collocation versus differential inclusion in direct optimization, *J. Guidance Contr. Dyn.*, 21, 5, 780-785 (1998) · [doi:10.2514/2.4306](#)

- [14] An, DTV; Yen, ND, Differential stability of convex optimization problems under inclusion constraints, *Appl. Anal.*, 94, 108-128 (2015) · [Zbl 1309.49014](#) · [doi:10.1080/00036811.2014.890710](#)
- [15] Giannessi, F.; Maugeri, A., *Variational Analysis and Applications* (2005), Berlin: Springer, Berlin · [Zbl 1077.49001](#) · [doi:10.1007/b105059](#)
- [16] Ioffe, A.D., Tikhomirov, V.: *Theory of extremal problems*. "Nauka". Moscow (1974) English transl. North-Holland. Amsterdam (1978)
- [17] Jeyakumar, V.; Wu, ZY, A qualification free sequential Pshenichnyi-Rockafellar lemma and convex semidefinite, *J. Convex Anal.*, 13, 3, 773-784 (2006) · [Zbl 1109.49015](#)
- [18] Khan, AA; Tammer, C.; Zalinescu, C., *Set-Valued Optimization* (2016), Berlin: Springer, Berlin · [Zbl 1308.49004](#)
- [19] Kourogenis, NC, Strongly nonlinear second order differential inclusions with generalized boundary conditions, *J. Math. Anal. Appl.*, 287, 348-364 (2003) · [Zbl 1046.34025](#) · [doi:10.1016/S0022-247X\(02\)00511-5](#)
- [20] Loewen, PD; Rockafellar, RT, Bolza problems with general time constraints, *SIAM J. Contr. Optim.*, 35, 2050-2069 (1997) · [Zbl 0904.49014](#) · [doi:10.1137/S0363012996298801](#)
- [21] Mahmudov, EN, Optimization of Mayer problem with Sturm-Liouville-type differential inclusions, *J. Optim. Theory Appl.*, 177, 2, 345-375 (2018) · [Zbl 1397.49025](#) · [doi:10.1007/s10957-018-1260-2](#)
- [22] Mahmudov, EN, *Single Variable Differential and Integral Calculus* (2013), Paris: Springer, Paris · [Zbl 1270.26002](#) · [doi:10.2991/978-94-91216-86-2](#)
- [23] Mahmudov, EN; Pshenichnyi, BN, The optimality principle for discrete and differential inclusions of parabolic type with distributed parameters, and duality, *Russ. Acad. Sci. Izvestiya Math.*, 42, 2, 299 (1994) · [Zbl 0811.49016](#) · [doi:10.1070/IM1994v042n02ABEH001538](#)
- [24] Mahmudov, EN, Approximation and optimization of Darboux type differential inclusions with set-valued boundary conditions, *Optim. Lett.*, 7, 871-891 (2013) · [Zbl 1272.49036](#) · [doi:10.1007/s11590-012-0460-1](#)
- [25] Mahmudov, EN, *Approximation and Optimization of Discrete and Differential Inclusions* (2011), Boston: Elsevier, Boston · [Zbl 1235.65002](#)
- [26] Mahmudov, EN, Approximation and optimization of higher order discrete and differential inclusions, *Nonlin. Differ. Equ. Appl. NoDEA*, 21, 1-26 (2014) · [Zbl 1301.49056](#) · [doi:10.1007/s00030-013-0234-1](#)
- [27] Mahmudov, EN, Convex optimization of second order discrete and differential inclusions with inequality constraints, *J. Convex Anal.*, 25, 1-26 (2018) · [Zbl 1434.53080](#) · [doi:10.1007/978-3-319-71834-7\\_1](#)
- [28] Mahmudov, EN, Optimal control of second order delay-discrete and delay-differential inclusions with state constraints, *Evol. Equ. Contr. Theory*, 7, 3, 501-529 (2018) · [Zbl 1407.49023](#) · [doi:10.3934/eect.2018024](#)
- [29] Mahmudov, EN, Locally adjoint mappings and optimization of the first boundary value problem for hyperbolic type discrete and differential inclusions, *Nonlinear Anal.*, 67, 2966-2981 (2007) · [Zbl 1117.49023](#) · [doi:10.1016/j.na.2006.09.054](#)
- [30] Mahmudov, EN, Necessary and sufficient conditions for discrete and differential inclusions of elliptic type, *J. Math. Anal. Appl.*, 323, 768-789 (2006) · [Zbl 1139.49021](#) · [doi:10.1016/j.jmaa.2005.10.069](#)
- [31] Mahmudov, EN; Mardanov, MJ, On duality in optimal control problems with second-order differential inclusions and initial-point constraints, *Proc. Inst. Math. Mech. Natl. Acad. Sci. Azerb.*, 46, 115-128 (2020) · [Zbl 1454.49035](#)
- [32] Marco, L.; Murillo, JA, Lyapunov functions for second order differential inclusions: a viability approach, *J. Math. Anal. Appl.*, 262, 339-354 (2001) · [Zbl 0997.34054](#) · [doi:10.1006/jmaa.2001.7583](#)
- [33] Martinez-Legaz, JE; Thera, M., A convex representation of maximal monotone operators, *Spec. Issue Prof. Ky Fan. J. Nonlin. Convex Anal.*, 2, 243-247 (2001) · [Zbl 0999.47037](#)
- [34] Mordukhovich, BS; Nam, NM, *An Easy Path to Convex Analysis and Applications* (2014), San Rafael: Morgan & Claypool Publishers, San Rafael · [Zbl 1284.49002](#)
- [35] Mordukhovich, BS; Outrata, JV; Sarabi, ME, Full stability of locally optimal solutions in second-order cone programs, *SIAM J. Optim.*, 24, 4, 1581-1613 (2014) · [Zbl 1311.49062](#) · [doi:10.1137/130928637](#)
- [36] Mordukhovich, BS, Discrete approximations and refined Euler-Lagrange conditions for nonconvex differential inclusions, *SIAM J. Contr. Optim.*, 33, 882-915 (1995) · [Zbl 0844.49017](#) · [doi:10.1137/S0363012993245665](#)
- [37] Ying, G.; Xinmin, Y.; Jin, Y.; Hong, Y., Scalarizations and Lagrange multipliers for approximate solutions in the vector optimization problems with set-valued maps, *J. Ind. Manag. Optim.*, 11, 673-683 (2014) · [Zbl 1305.90365](#)
- [38] Zhang, Q.; Li, G., Nonlinear boundary value problems for second order differential inclusions, *Inter. J. Nonlinear Sci.*, 9, 84-103 (2010) · [Zbl 1213.34023](#)

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