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**Convergence rate of Euler-Maruyama scheme for SDDEs of neutral type.** (English)

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Summary: In this paper, we are concerned with the convergence rate of Euler-Maruyama (EM) scheme for stochastic differential delay equations (SDDEs) of *neutral type*, where the neutral, drift, and diffusion terms are allowed to be of polynomial growth. More precisely, for SDDEs of neutral type driven by Brownian motions, we reveal that the convergence rate of the corresponding EM scheme is one-half; Whereas for SDDEs of neutral type driven by pure jump processes, we show that the best convergence rate of the associated EM scheme is slower than one-half. As a result, the convergence rate of general SDDEs of neutral type, which is dominated by pure jump process, is slower than one-half.

**MSC:**

65C30 Numerical solutions to stochastic differential and integral equations

60H10 Stochastic ordinary differential equations (aspects of stochastic analysis)

**Keywords:**

stochastic differential delay equation of neutral type; polynomial condition; Euler scheme; convergence rate; jump processes

**Full Text:** DOI

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