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A unified asymptotic preserving and well-balanced scheme for the Euler system with multiscale relaxation. (English) Zbl 07464448

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Summary: The design and analysis of a unified asymptotic preserving (AP) and well-balanced scheme for the Euler Equations with gravitational and frictional source terms is presented in this paper. The asymptotic behaviour of the Euler system in the limit of zero Mach and Froude numbers, and large friction is characterised by an additional scaling parameter. Depending on the values of this parameter, the Euler system relaxes towards a hyperbolic or a parabolic limit equation. Standard Implicit-Explicit Runge-Kutta schemes are incapable of switching between these asymptotic regimes. We propose a time semi-discretisation to obtain a unified scheme which is AP for the two different limits. A further reformulation of the semi-implicit scheme can be recast as a fully-explicit method in which the mass update contains both hyperbolic and parabolic fluxes. A space-time fully-discrete scheme is derived using a finite volume framework. A hydrostatic reconstruction strategy, an upwinding of the sources at the interfaces, and a careful choice of the central discretisation of the parabolic fluxes are used to achieve the well-balancing property for hydrostatic steady states. Results of several numerical case studies are presented to substantiate the theoretical claims and to verify the robustness of the scheme.

MSC:

35L45 Initial value problems for first-order hyperbolic systems
35L60 First-order nonlinear hyperbolic equations
35L65 Hyperbolic conservation laws
35L67 Shocks and singularities for hyperbolic equations
65M06 Finite difference methods for initial value and initial-boundary value problems involving PDEs
65M08 Finite volume methods for initial value and initial-boundary value problems involving PDEs
76-XX Fluid mechanics

Keywords:

compressible Euler system; multiscale relaxation; unified asymptotic preserving; finite volume method; hydrostatic steady states; well-balancing

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