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A variational model of elasto-plastic behavior of materials. (English) Zbl 07464181

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Summary: We present a mesoscale phase field model of the elasto-plastic behavior of materials as an extension of the model of plastic slip put forth by Ambrosio et al. (J. Elast. 110:201-235, 2013). In the proposed model, we consider a new strain energy density decomposition based on the degradation of the shear modulus of the material and a new formulation of the surface plastic energy term that accounts for the energy that is consumed when gliding occurs across slip bands. The resulting constitutive model leads to the introduction of an additional strain contribution that we term the gliding strain tensor. With this formulation, the phase field distribution captures the nature of the activation process of slip planes and is capable of characterizing a wide range of plastic behavior, from very diffuse to highly localized, as evidenced by several two-dimensional numerical results.

Our results also show that this mesoscopic model is able to capture the Hall-Petch effect, wherein the strength of some polycrystalline materials is inversely proportional to the square root of the average grain size. This means that the size effect does not necessarily need to be explained with models at microscopic scale but can be captured by mesoscopic models. The proposed model is also consistent with the classical theory of plasticity in terms of the postulate of incompressibility and the von Mises yield criterion.

MSC:

35Qxx Partial differential equations of mathematical physics and other areas of application
35R35 Free boundary problems for PDEs
74C05 Small-strain, rate-independent theories of plasticity (including rigid-plastic and elasto-plastic materials)
74G65 Energy minimization in equilibrium problems in solid mechanics
74R20 Anelastic fracture and damage

Keywords:

variational approach; free discontinuity problem; plasticity; phase field method; shear band

Software:

FEniCS

Full Text: DOI

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