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A thermodynamic approach to rate-type models of elastic-plastic materials. (English)

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The objective of the paper is to derive rate-independent models, which are either non-dissipative, or dissipative and compatible with the second law of thermodynamics. Under the supposition that the Helmholtz (or Gibbs) free energy and the product of entropy, called the rate of dissipation (denoted by Γ, with $\Gamma \geq 0$), are constitutive functions dependent on a certain set of independent variables, the second law of thermodynamics is reduced to the appropriate dissipation equalities in Section 3 and 4. The dissipation equalities (9) and (14) are written in terms of symmetric Piola Kirchhoff, T_{RR}, Green-St. Venant deformation tensor, E, and their time derivative, with respect to the reference configuration. By passing to the Eulerian description, the dissipation equalities are rewritten under the form (10) and (15) in terms of Cauchy stress, Eulerian Almansi strain tensor, and their objective derivatives, namely Truesdell stress derivative and stretching tensor, respectively. In Section 5 the three dimensional, finite deformation models are derived for special selections of Γ, namely Γ depends either on $\frac{d}{dt}E$, $\frac{d}{dt}T_{RR}$, via their norms, or on stress power $T_{RR}\cdot\frac{d}{dt}E$, with appropriate rate-independent non-negative, proportional factors. When these factors are vanishing the non-dissipative models follow. Based on the decomposition of a second order tensor given in Proposition 1, the Truesdell hypo-elastic type, rate-independent, but non-dissipative models are provided in the reference configuration, (20), while their counterpart, (24), written in the spatial description, expresses the Truesdell stress derivative as linear tensor function with respect to the stretching tensor. The rate-independent constitutive equations (23) and (25) are associated with general dissipative equations using the same decomposition result. The mentioned constitutive equations are derived under the hypothesis that the free energy essentially depends on stress tensor. The appropriate fourth-order tensor valued functions are dependent on the stress and strain measures and contain parts which do not generate power, as a consequence of the decomposition given in Proposition 1. The one-dimensional models, proposed in Sections 6–11, describe the material response either to cyclic strain-driven or to cyclic stress-driven processes. The irreversible material behaviour of Duhem like solids (as the authors called them) is characterized in terms of the stress induced by a given appropriate deformation process, or of the strain generated by a certain given stress process, respectively. The proposed models are compared with elastic-plastic models existing in the literature.

Reviewer: Sanda Cleja-Ţigoiu (Bucureşti)

MSC:

74C15 Large-strain, rate-independent theories of plasticity (including nonlinear plasticity)
74A15 Thermodynamics in solid mechanics
74A20 Theory of constitutive functions in solid mechanics

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