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A discrete velocity numerical scheme for the two-dimensional bitemperature Euler system.

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Authors' abstract: This paper is devoted to the numerical approximation of the bidimensional bitemperature Euler system. This model is a nonconservative hyperbolic system describing an out of equilibrium plasma in a quasi-neutral regime, with applications in inertial confinement fusion. One main difficulty here is to handle shock solutions involving the product of the velocity by pressure gradients. The authors develop a second order numerical scheme by using a discrete BGK relaxation model. The second order extension is based on a subdivision of each cartesian cell into four triangles to perform affine reconstructions of the solution. Such ideas have been developed in the literature for systems of conservation laws. Here, it is shown how they can be used in a nonconservative setting. The numerical method is implemented and tested in the last part of the paper.

Reviewer: Victor Michel-Dansac (Strasbourg)

MSC:

65M08 Finite volume methods for initial value and initial-boundary value problems involving PDEs
65M06 Finite difference methods for initial value and initial-boundary value problems involving PDEs
65N08 Finite volume methods for boundary value problems involving PDEs
35L60 First-order nonlinear hyperbolic equations
76X05 Ionized gas flow in electromagnetic fields; plasmic flow
76L05 Shock waves and blast waves in fluid mechanics
35Q31 Euler equations
82C40 Kinetic theory of gases in time-dependent statistical mechanics
82D75 Nuclear reactor theory; neutron transport

Keywords:

nonconservative hyperbolic system; Euler-type model for plasmas; discrete BGK approximation; second order

Software:

HE-E1GODF

Full Text: DOI

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