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Optimal decay rates of solutions to hyperbolic conservation laws with damping. (English)

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Summary: In this paper, we are concerned with the asymptotic behavior of solutions to the system of hyperbolic conservation laws with damping. In particular, a system includes compressible Euler equations with damping, M_1-model, etc. Under some smallness conditions on initial perturbations, we prove that the solutions to the Cauchy problem of the system globally exist and time-asymptotically converge to corresponding equilibrium state, and further give the optimal convergence rate. The approach adopted is the technical time-weighted energy method combined with the Green's function method.

MSC:

85A25 Radiative transfer in astronomy and astrophysics
35L65 Hyperbolic conservation laws
35B40 Asymptotic behavior of solutions to PDEs
35Q05 Euler-Poisson-Darboux equations
65B15 Euler-Maclaurin formula in numerical analysis
58J47 Propagation of singularities; initial value problems on manifolds
35J08 Green's functions for elliptic equations
83C40 Gravitational energy and conservation laws; groups of motions

Keywords:

Euler equations with damping; M_1-model; time-weighted energy method; optimal convergence rate

Full Text: DOI

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