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Sonic-supersonic solutions to a degenerate Cauchy-Goursat problem for 2D relativistic Euler equations. (English) Zbl 07461638

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Summary: In this paper, we consider the degenerate Cauchy-Goursat problem for 2D steady isentropic relativistic Euler equations. Prescribing the sonic curve and a positive characteristic curve as boundaries, the existence and uniqueness of sonic-supersonic solution in an angular region are obtained. Employing the characteristic decomposition of angle variables, 2D relativistic Euler equations are transformed into the first-order hyperbolic equations. In the partial hodograph plane, introducing the change variables $\overline{W} = \frac{1}{W}, \overline{Z} = \frac{-1}{Z}$, associated with the iterative method in Li, Hu (2019) yields a linear equations and the existence and uniqueness of the smooth sonic-supersonic solutions are established. Finally, we return the solution in the partial hodograph plane to that in the original physical variables.

MSC:

35Q31 Euler equations
35L65 Hyperbolic conservation laws
35L67 Shocks and singularities for hyperbolic equations
76N15 Gas dynamics (general theory)

Keywords:

relativistic Euler equations; characteristic decomposition; degenerate Cauchy-Goursat problem; sonic-supersonic solutions

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