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**Wu, Qiwei**; **Li, Yeping**; **Xu, Rui**

**Large-time behavior of solutions to bipolar Euler-Poisson equations with time-dependent damping in the half space.** (English) Zbl 07461315

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Summary: This paper is concerned with the large-time behavior of solutions to an initial boundary value problem for the one-dimensional bipolar Euler-Poisson equations with time-dependent damping effects $\frac{J_i}{(1+t)^\lambda}(i=1,2)$ for $-1<\lambda<1$. We first show the decay rates of the corresponding asymptotic profiles, the so-called nonlinear diffusion waves, then by means of the time-weighted energy method, we prove that the smooth solutions to the initial-boundary value problem exist uniquely and globally, and time-asymptotically converge to the nonlinear diffusion waves, provided that the initial perturbation around the diffusion wave is small enough. The convergence rates are in the forms that $O(t^{-\frac{3}{4}(1+\lambda)})$ for $-1<\lambda<\frac{3}{5}$ and $O(t^{\frac{\lambda-3}{2}})$ for $\frac{3}{5}<\lambda<1$, respectively, where $\lambda=\frac{3}{5}$ is the critical point, and the convergence rate at the critical point is $O(t^{\frac{6}{5}}\ln t)$. The results are different from those of the Cauchy problem in Li et al. (2019) [20].

**MSC:**

35Qxx Partial differential equations of mathematical physics and other areas of application

35Bxx Qualitative properties of solutions to partial differential equations

82Dxx Applications of statistical mechanics to specific types of physical systems

**Keywords:**

bipolar Euler-Poisson equations; nonlinear diffusion waves; time-dependent damping; large-time behavior; initial-boundary value problem

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