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Entropic regularization of the discontinuous Galerkin method in conservative variables for two-dimensional Euler equations. (Russian. English summary) [Zbl 07461140](#)

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Summary: The entropic regularization of the conservative stable discontinuous Galerkin method in conservative variables is constructed on the basis of a special slope limiter for the twodimensional Euler equations. This limiter ensures the fulfillment of the two-dimensional analogs of the monotonicity conditions and a discrete analog of the entropy inequality. The developed method was tested on two-dimensional model gas-dynamic problems.

MSC:

76M10 Finite element methods applied to problems in fluid mechanics

76M20 Finite difference methods applied to problems in fluid mechanics

76N15 Gas dynamics (general theory)

Keywords:

Euler equations; discontinuous Galerkin method; finite difference time discretization; conservation law; slope limiter; entropic inequality

Software:

[FLEXI](#)

Full Text: [DOI](#) [MNR](#)

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