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A morawetz inequality for gravity-capillary water waves at low bond number. (English)

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Summary: This paper is devoted to the 2D gravity-capillary water waves equations in their Hamiltonian formulation, addressing the general question of proving Morawetz inequalities. We continue the analysis initiated in our previous work, where we have established local energy decay estimates for gravity waves. Here we add surface tension and prove a stronger estimate with a local regularity gain, akin to the smoothing effect for dispersive equations. Our main result holds globally in time and holds for genuinely nonlinear waves, since we are only assuming some very mild uniform Sobolev bounds for the solutions. Furthermore, it is uniform both in the infinite depth limit and the zero surface tension limit.

MSC:

- 76B15 Water waves, gravity waves; dispersion and scattering, nonlinear interaction
- 35Q35 PDEs in connection with fluid mechanics
- 35R35 Free boundary problems for PDEs
- 76B45 Capillarity (surface tension) for incompressible inviscid fluids
- 35Q31 Euler equations

Keywords:

water waves; gravity/capillary; local energy decay

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