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Uniqueness and energy balance for isentropic Euler equation with stochastic forcing. (English) Zbl 1477.35144

Nonlinear Anal., Real World Appl. 61, Article ID 103328, 18 p. (2021).

Summary: In this article, we prove uniqueness and energy balance for isentropic Euler system driven by a cylindrical Wiener process. Pathwise uniqueness result is obtained for weak solutions having Hölder regularity C^α, $\alpha > 1/2$ in space and satisfying one-sided Lipschitz bound on velocity. We prove Onsager's conjecture for isentropic Euler system with stochastic forcing, that is, energy balance equation for solutions enjoying Hölder regularity C^α, $\alpha > 1/3$. Both the results have been obtained in a more general setting by considering regularity in Besov space.

MSC:

35Q31 Euler equations
76N10 Existence, uniqueness, and regularity theory for compressible fluids and gas dynamics
35D30 Weak solutions to PDEs
35B65 Smoothness and regularity of solutions to PDEs

Keywords:

stochastic isentropic Euler system; pathwise weak solution; uniqueness; Besov space; energy balance; Onsager's conjecture

Full Text: DOI arXiv

References:

[1] Feireisl, E.; Ghoshal, S. S.; Jana, A., On uniqueness of dissipative solutions to the isentropic Euler system, Commun. Partial. Differ. Equ., 44, 12, 1285-1298 (2019) · Zbl 1428.35325

[2] Berthelin, F.; Vovelle, J., Stochastic isentropic Euler equations, Ann. Sci. de L'ecole Normale Sup., 52, 1, 181-254 (2019) · Zbl 1470.60177

[3] Breit, D.; Feireisl, E.; Hofmanová, M., On solvability and ill-posedness of the compressible Euler system subject to stochastic forces, Anal. PDE, 13, 2, 371-402 (2020) · Zbl 1435.35289

[4] Kim, J. U., On the stochastic quasi-linear symmetric hyperbolic system, J. Differential Equations, 250, 3, 1650-1684 (2011) · Zbl 1211.35174

[5] Breit, D.; Mensah, P. R., Stochastic compressible Euler equations and inviscid limits, Nonlinear Anal., 184, 218-238 (2019) · Zbl 1482.35281

[6] Breit, D.; Hofmanová, M., Stochastic Navier-Stokes equations for compressible fluids, Indiana Univ. Math. J., 65, 4, 1183-1250 (2016) · Zbl 1358.60072

[7] Breit, D.; Feireisl, E.; Hofmanová, M., Local strong solutions to the stochastic compressible Navier-Stokes system, Commun. Partial. Differ. Equ., 43, 313-345 (2018) · Zbl 1390.60226

[8] Breit, D.; Feireisl, E.; Hofmanová, M., Compressible fluids driven by stochastic forcing: The relative energy inequality and applications, Comm. Math. Phys., 350, 443-473 (2017) · Zbl 1362.35228

[9] Breit, D.; Feireisl, E.; Hofmanová, M.; Maslowski, B., Stationary solutions to the compressible Navier-Stokes system driven by stochastic forces, Probab. Theory Related Fields, 174, 3-4, 981-1032 (2019) · Zbl 1447.35241

[10] Dafermos, C. M., The second law of thermodynamics and stability, Arch. Ration. Mech. Anal., 70, 2, 167-179 (1979) · Zbl 0448.73004

[11] DiPerna, R. J., Uniqueness of solutions to hyperbolic conservation laws, Indiana Univ. Math. J., 28, 1, 137-188 (1979) · Zbl 0409.35057

[12] Chen, G.-Q.; Frid, H.; Li, Y., Uniqueness and stability of Riemann solutions with large oscillation in gas dynamics, Comm. Math. Phys., 228, 2, 201-217 (2002) · Zbl 1029.76045

[13] Feireisl, E.; Kreml, O., Uniqueness of rarefaction waves in multidimensional compressible Euler system, J. Hyperbolic Differ. Eq., 12, 3, 489-499 (2015) · Zbl 1327.35230

[14] Wiedemann, E., Weak-Strong Uniqueness in Fluid Dynamics (2018), Cambridge University Press: Cambridge University Press 289-326 · Zbl 1408.35158

[15] Chiodaroli, E.; De Lellis, C.; Kreml, O., Global ill-posedness of the isentropic system of gas dynamics, Comm. Pure Appl.

Math., 68, 7, 1157-1190 (2015) · Zbl 1323.35137

[16] De Lellis, C.; Székelyhidi, L., The Euler equations as a differential inclusion, Ann. of Math. (2), 170, 3, 1417-1436 (2009) · Zbl 1350.35146

[17] Feireisl, E., Weak solutions to problems involving inviscid fluids, (Shibata, Y., Mathematical Fluid Dynamics, Present and Future, Vol. 183. Mathematical Fluid Dynamics, Present and Future, Vol. 183, Springer Proceedings in Mathematics and Statistics (2016), Springer: Springer New York), 377-399 · Zbl 1371.35204

[18] Ghoshal, S. S.; Jana, A., Uniqueness of dissipative solutions to the complete Euler system, J. Math. Fluid Mech., 23, 2, 34 (2021) · Zbl 1460.35271

[19] Ghoshal, S. S.; Jana, A.; Koumatos, K., On the uniqueness of solutions to hyperbolic systems of conservation laws (2020), arXiv Preprint, arXiv:2007.10923v1

[20] Onsager, L., Statistical hydrodynamics, Nuovo Cimento, 6, 279 (1949), (Supplemento)

[21] Constantin, P.; Titi, E. S., Onsager's conjecture on the energy conservation for solutions of Euler's equation, Comm. Math. Phys., 165, 1, 207-209 (1994) · Zbl 0818.35085

[22] Fjordholm, U. S.; Wiedemann, E., Statistical solutions and Onsager's conjecture, Physica D, 376, 259-265 (2018) · Zbl 1398.35153

[23] Akramov, I.; Debiec, T.; Skipper, J.; Wiedemann, E., Energy conservation for the compressible Euler and Navier-Stokes equations with vacuum, Anal. PDE, 13, 3, 789-811 (2020) · Zbl 1437.35552

[24] Eyink, G. L., Energy dissipation without viscosity in ideal hydrodynamics. I. Fourier analysis and local energy transfer, Physica D, 78, 3-4, 222-240 (1994) · Zbl 0817.76011

[25] Feireisl, E.; Gwiazda, P.; Świerczewska-Gwiazda, A.; Wiedemann, E., Regularity and energy conservation for the compressible Euler equations, Arch. Ration. Mech. Anal., 223, 3, 1-21 (2017) · Zbl 1365.35113

[26] Bardos, C.; Gwiazda, P.; Świerczewska-Gwiazda, A.; Titi, E. S.; Wiedemann, E., Onsager's conjectutre in bounded domains for the conservation of entropy and other companion laws, Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci., 475, Article 20190289 pp. (2019)

[27] Bardos, C.; Gwiazda, P.; Świerczewska-Gwiazda, A.; Titi, E. S.; Wiedemann, E., On the extension of Onsager's conjecture for general conservation laws, J. Nonlinear Sci., 29, 501-510 (2019) · Zbl 1420.35202

[28] Breit, D.; Feireisl, E.; Hofmanová, M., (Stochastically Forced Compressible Fluid Flows. Stochastically Forced Compressible Fluid Flows, De Gruyter Series in Applied and Numerical Mathematics, vol. 3 (2018)) · Zbl 1387.76001

[29] Gawarecki, L.; Mandrekar, V., Stochastic Differential Equations in Infinite Dimensions with Applications To Stochastic Partial Differential Equations, Probability and Its Applications (2011), Springer: Springer New York · Zbl 1228.60002

[30] Gwiazda, P.; Michálek, M.; Świerczewska-Gwiazda, A., A note on weak solutions of conservation laws and energy/entropy conservation, Arch. Ration. Mech. Anal., 229, 3, 1223-1238 (2018) · Zbl 1398.35168

[31] Itô, K., Foundations of Stochastic Differential Equations in Infinite-Dimensional Spaces CBMS-NSF Regional Conference Series in Applied Mathematics, Vol. 47 (1984), Society for Industrial and Applied Mathematics (SIAM): Society for Industrial and Applied Mathematics (SIAM) Philadelphia, PA

[32] Da Prato, G.; Zabczyk, J., Stochastic Equations in Infinite Dimensions Encyclopedia of Mathematics and Its Applications, Vol. 152 (2014), Cambridge University Press: Cambridge University Press Cambridge