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The strong convergence and stability of explicit approximations for nonlinear stochastic delay differential equations. (English) Zbl 1481.65023

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Summary: This paper focuses on explicit approximations for nonlinear stochastic delay differential equations (SDDEs). Under less restrictive conditions, the truncated Euler-Maruyama (TEM) schemes for SDDEs are proposed, which numerical solutions are bounded in the qth moment for $q \geq 2$ and converge to the exact solutions strongly in any finite interval. The $1/2$ order convergence rate is yielded. Furthermore, the long-time asymptotic behaviors of numerical solutions, such as stability in mean square and $\mathbb{P}-1$, are examined. Several numerical experiments are carried out to illustrate our results.

MSC:

65C30 Numerical solutions to stochastic differential and integral equations
34K50 Stochastic functional-differential equations
60H10 Stochastic ordinary differential equations (aspects of stochastic analysis)
60H35 Computational methods for stochastic equations (aspects of stochastic analysis)
65L20 Stability and convergence of numerical methods for ordinary differential equations

Keywords:

stochastic delay differential equations; explicit truncated Euler-Maruyama scheme; moment bound; strong convergence; stability analysis

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