Kelly, Cónall; Lord, Gabriel J.

Adaptive Euler methods for stochastic systems with non-globally Lipschitz coefficients. (English) Zbl 1480.60182

Numer. Algorithms 89, No. 2, 721-747 (2022).

Summary: We present strongly convergent explicit and semi-implicit adaptive numerical schemes for systems of semi-linear stochastic differential equations (SDEs) where both the drift and diffusion are not globally Lipschitz continuous. Numerical instability may arise either from the stiffness of the linear operator or from the perturbation of the nonlinear drift under discretization, or both. Typical applications arise from the space discretization of an SPDE, stochastic volatility models in finance, or certain ecological models. Under conditions that include montonicity, we prove that a timestepping strategy which adapts the stepsize based on the drift alone is sufficient to control growth and to obtain strong convergence with polynomial order. The order of strong convergence of our scheme is $(1-\epsilon)/2$, for $\epsilon \in (0,1)$, where ϵ becomes arbitrarily small as the number of finite moments available for solutions of the SDE increases. Numerically, we compare the adaptive semi-implicit method to a fully drift-implicit method and to three other explicit methods. Our numerical results show that overall the adaptive semi-implicit method is robust, efficient, and well suited as a general purpose solver.

MSC:

60H15 Stochastic partial differential equations (aspects of stochastic analysis)
60H35 Computational methods for stochastic equations (aspects of stochastic analysis)
65C30 Numerical solutions to stochastic differential and integral equations
91B70 Stochastic models in economics

Keywords:

stochastic differential equations; adaptive timestepping; semi-implicit Euler method; non-globally Lipschitz coefficients; strong convergence

Full Text: DOI arXiv

References:

[1] Appleby, J.; Kelly, C.; Mao, X.; Rodkina, A., On the local dynamics of polynomial difference equations with fading stochastic perturbations, Dyn. Contin. Discrete Impuls. Syst. Ser. A Math. Anal., 17, 401-430 (2010) · Zbl 1192.39016

[2] Beyn, W-J; Isaak, E.; Kruse, R., Stochastic c-stability and b-consistency of explicit and implicit euler-type schemes, J. Sci. Comput., 67, 3, 955-987 (2016) · Zbl 1362.65011 · doi:10.1007/s10915-015-0114-4

[3] Buckwar, E.; Kelly, C., Non-normal drift structures and linear stability analysis of numerical methods for systems of stochastic differential equations, Comput. Math. Appl., 64, 2282-2293 (2012) · Zbl 1268.60091 · doi:10.1016/j.camwa.2012.02.059

[4] Burrage, PM; Burrage, K., A variable stepsize implementation for stochastic differential equations, SIAM J. Sci. Comput., 24, 3, 848-864 (2002) · Zbl 1034.65003 · doi:10.1137/S1064827500376922

[5] Gains, JG; Lyons, TJ, Variable step size control in the numerical solution of stochastic differential equations, SIAM J. Appl. Math., 57, 1455-1484 (1997) · Zbl 0888.60046 · doi:10.1137/S0036139995286515

[6] Fang, W.; Giles, M., Adaptive Euler-Maruyama method for SDEs with non-globally Lipschitz drift, Ann. Appl. Probab., 30, 2, 526-560 (2020) · Zbl 1464.60061 · doi:10.1214/19-AAP1507

[7] Higham, DJ; Mao, X.; Stuart, AM, Strong convergence of Euler-type methods for nonlinear stochastic differential equations, SIAM J. Numer. Anal., 40, 1041-1063 (2002) · Zbl 1026.65003 · doi:10.1137/S0036142901389530

[8] Higham, DJ; Trefethen, LN, Stiffness of ODEs, BIT Numer. Math., 33, 2, 285-303 (1993) · Zbl 0782.65091 · doi:10.1007/BF01989751

[9] Hutzenthaler, M.; Jentzen, A., Numerical approximations of stochastic differential equations with non-globally Lipschitz continuous coefficients, Mem. Amer. Math. Soc., 236, v + 99 (2015) · Zbl 1330.60084

[10] Hutzenthaler, M., Jentzen, A.: On a perturbation theory and on strong convergence rates for stochastic ordinary and partial differential equations with non-globally monotone coefficients. arXiv:1401.0295 · Zbl 07206753

[11] Hutzenthaler, M.; Jentzen, A.; Kloeden, PE, Strong and weak divergence in finite time of Euler's method for stochastic differential equations with non-globally Lipschitz continuous coefficients, Proc. R. Soc. Lond. Ser. A Math. Phys. Eng. Sci., 467, 1563-1576 (2011) · Zbl 1228.65014

[12] Hutzenthaler, M.; Jentzen, A.; Kloeden, PE, Strong convergence of an explicit numerical method for SDEs with nonglobally Lipschitz continuous coefficients, Ann. Appl. Probab., 22, 1611-1641 (2012) · Zbl 1256.65003 · doi:10.1214/11-AAP803

[13] Ilie, S.; Jackson, KR; Enright, WH, Adaptive time-stepping for the strong numerical solution of stochastic differential equations, Numer. Algor., 68, 791-812 (2015) · Zbl 1312.65006 · doi:10.1007/s11075-014-9872-6

[14] Kelly, C.; Lord, GJ, Adaptive timestepping strategies for nonlinear stochastic systems, IMA J. Numer. Anal., 38, 3, 1523-1549 (2018) · Zbl 1477.65023 · doi:10.1093/imanum/drx036

[15] Kelly, C., Lord, G.J., Sun, F.: Strong convergence of an adaptive time-stepping Milstein method for SDEs with one-sided Lipschitz drift. arXiv:1909.00099

[16] Kelly, C.; Rodkina, A.; Rapoo, A., Adaptive timestepping for pathwise stability and positivity of strongly discretised nonlinear stochastic differential equations, J. Comput. Appl. Math., 334, 39-57 (2018) · Zbl 1385.37070 · doi:10.1016/j.cam.2017.11.027

[17] Lewis, A.L.: Option Valuation under Stochastic Volatility: with Mathematica Code. Finance Pres (2000) · Zbl 0937.91060

[18] Liu, W.; Mao, X., Almost sure stability of the Euler-Maruyama method with random variable stepsize for stochastic differential equations, Numer. Algor., 74, 2, 573-592 (2017) · Zbl 1371.65010 · doi:10.1007/s11075-016-0162-3

[19] Lord, G.J., Campbell, S.: Adaptive time-stepping for Stochastic Partial Differential Equations with non-Lipschitz drift. arXiv:1812.09036

[20] Lord, GJ; Powell, CE; Shardlow, T., An introduction to computational stochastic PDEs, Cambridge Texts in Applied Mathematics (2014), New York: Cambridge University Press, New York · Zbl 1327.60011 · doi:10.1017/CBO9781139017329

[21] Mao, X., Stochastic Differential Equations and Applications (2008), Horwood: 2, Horwood· doi:10.1533/9780857099402

[22] Mao, X., The truncated Euler-Maruyama method for stochastic differential equations, J. Comput. Appl. Math., 290, 370-384 (2015) · Zbl 1330.65016 · doi:10.1016/j.cam.2015.06.002

[23] Mao, X., Convergence rates of the truncated Euler-Maruyama method for stochastic differential equations, J. Comput. Appl. Math., 296, 362-375 (2016) · Zbl 1378.65036 · doi:10.1016/j.cam.2015.09.035

[24] Mao, X.; Szpruch, L., Strong convergence and stability of implicit numerical methods for stochastic differential equations with non-globally Lipschitz continuous coefficients, J. Comput. Appl. Math., 238, 14-28 (2013) · Zbl 1262.65012 · doi:10.1016/j.cam.2012.08.015

[25] Sabanis, S., A note on tamed Euler approximations, Electron, Commun. Probab., 18, 10 (2013) · Zbl 1329.60237

[26] Sabanis, S., Euler approximations with varying coefficients: the case of superlinearly growing diffusion coefficients,, Ann. Appl. Probab., 26, 2083-2105 (2016) · Zbl 1352.60101 · doi:10.1214/15-AAP1140

[27] Shiryaev, AN, Probability (1996), Berlin: Springer, Berlin· doi:10.1007/978-1-4757-2539-1

[28] Stuart, A.M., Humphries, A.R.: Dynamical Systems and Numerical Analysis CUP (1996) · Zbl 0869.65043

[29] Szpruch, L., Zhang, X.: V-integrability, asymptotic stability and comparison theorem of explicit numerical schemes for SDEs. Math. Comput. To appear (2018) · Zbl 1380.65020

[30] Tretyakov, MV; Zhang, Z., A fundamental mean-square convergence theorem for SDEs with locally Lipschitz coefficients and its applications, SIAM J. Numer. Anal., 51, 3135-3162 (2013) · Zbl 1293.60069 · doi:10.1137/120902318

[31] Zhang, Z.; Ma, H., Order-preserving strong schemes for SDEs with locally Lipschitz coefficients, Appl. Numer. Math., 112, 1-16 (2017) · Zbl 1354.65017 · doi:10.1016/j.apnum.2016.09.013