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**On vanishing near corners of conductive transmission eigenfunctions.** (English)

Zbl 1481.35301

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**Summary:** This paper is concerned with the geometric structure of the transmission eigenvalue problem associated with a general conductive transmission condition. We prove that under a mild regularity condition in terms of the Herglotz approximations of one of the pair of the transmission eigenfunctions, the eigenfunctions must be vanishing around a corner on the boundary. The Herglotz approximation is the Fourier extension of the transmission eigenfunction, and the growth rate of the density function can be used to characterize the regularity of the underlying wave function. The geometric structures derived in this paper include the related results in [*H. Diao et al.*, Commun. Partial Differ. Equations 46, No. 4, 630–679 (2021; Zbl 1475.35328)] and [*E. Blåsten and the third author*, J. Funct. Anal. 273, No. 11, 3616–3632 (2017; Zbl 1387.35437)] as special cases and verify that the vanishing around corners is a generic local geometric property of the transmission eigenfunctions.

**MSC:**

35P25 Scattering theory for PDEs

35J57 Boundary value problems for second-order elliptic systems

35R30 Inverse problems for PDEs

58J05 Elliptic equations on manifolds, general theory

78A05 Geometric optics

Cited in **3** Documents

**Keywords:**

conductive transmission eigenfunctions; corner singularity; geometric structures; vanishing; Herglotz approximation

**Full Text:** [DOI](#) [arXiv](#)

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