Gaburro, Elena; Castro, Manuel J.; Dumbser, Michael
A well balanced finite volume scheme for general relativity. (English) Zbl 07456293
SIAM J. Sci. Comput. 43, No. 6, B1226-B1251 (2021)

MSC:

65-XX Numerical analysis
35L40 First-order hyperbolic systems
65M08 Finite volume methods for initial value and initial-boundary value problems involving PDEs
83C05 Einstein's equations (general structure, canonical formalism, Cauchy problems)
83C10 Equations of motion in general relativity and gravitational theory
85-08 Computational methods for problems pertaining to astronomy and astrophysics
85-10 Mathematical modeling or simulation for problems pertaining to astronomy and astrophysics
85A30 Hydrodynamic and hydromagnetic problems in astronomy and astrophysics

Keywords:

first order hyperbolic systems; finite volume schemes; well balanced schemes; Einstein-Euler system; general relativity; first order conformal and covariant reformulation of the Einstein field equations

Software:

WhiskyMHD; Cosmos++

Full Text: DOI arXiv

References:

[1] M. Alcubierre, Introduction to 3+1 Numerical Relativity, Internat. Ser. Mongr. Phys. 140, Oxford University Press, 2008. · Zbl 1140.83002

[2] D. Alic, C. Bona, and C. Bona-Casas, Towards a gauge-polyvalent numerical relativity code, Phys. Rev. D, 79 (2009), 044026.

[3] D. Alic, C. Bona-Casas, C. Bona, L. Rezzolla, and C. Palenzuela, Conformal and covariant formulation of the Z4 system with constraint-violation damping, Phys. Rev. D, 85 (2012), 064040.

[4] D. Alic, W. Kastaun, and L. Rezzolla, Constraint damping of the conformal and covariant formulation of the Z4 system in simulations of binary neutron stars, Phys. Rev. D, 88 (2013), 064049.

[5] M.-Á. Aloy and I. Cordero-Carrión, Minimally implicit Runge-Kutta methods for Resistive Relativistic MHD, J. Phys.: Conf. Ser., 719 (2015), 2016, 012015.

[6] M. A. Aloy, J. M. Ibánez, J. M. Martí, and E. Müller, GENESIS: A high-resolution code for three-dimensional relativistic hydrodynamics, Astrophys. J. Suppl. Ser., 122 (1999), pp. 151-166.

[7] M. Anderson, E. W. Hirschmann, L. Lehner, S. L. Liebling, P. M. Motl, D. Neilsen, C. Palenzuela, and J. E. Tohline, Magnetized neutron-star mergers and gravitational-wave signals, Phys. Rev. Lett., 100 (2008), 191101.

[8] A. M. Anile, Relativistic Fluids and Magneto-fluids: With Applications in Astrophysics and Plasma Physics, Cambridge University Press, Cambridge, UK, 1990. · Zbl 0701.76003

[9] A. M. Anile, J. C. Miller, and S. Motta, Formation and damping of relativistic strong shocks, Phys. Fluids, 26 (1983), pp. 1450-1460. · Zbl 0528.76127

[10] P. Anninos, P. C. Fragile, and J. D. Salmonson, Cosmos++: Relativistic magnetohydrodynamics on unstructured grids with local adaptive refinement, Astrophys. J., 635 (2005), pp. 723-740.

[11] L. Antón, O. Zanotti, J. A. Miralles, J. M. Martí, J. M. Ibán⊠ez, J. A. Font, and J. A. Pons, Numerical 3+1 general relativistic magnetohydrodynamics: A local characteristic approach, Astrophys. J., 637 (2006), pp. 296-312.

[12] R. Arnowitt, S. Deser, and C. W. Misner, The dynamics of general relativity, in Gravitation: An Introduction to Current Research, John Wiley \& Sons, New York, 1962, pp. 227-265. · Zbl 1152.83320

[13] R. Arnowitt, S. Deser, and C. W. Misner, Republication of: The dynamics of general relativity, Gen. Relativ. Gravit., 40 (2008), pp. 1997-2027. · Zbl 1152.83320

[14] L. Arpaia and M. Ricchiuto, Well balanced residual distribution for the ALE spherical shallow water equations on moving adaptive meshes, J. Comput. Phys., 405 (2020), 109173. · Zbl 1453.65295

[15] E. Audusse, F. Bouchut, M.-O. Bristeau, R. Klein, and B. Perthame, A fast and stable well-balanced scheme with hydrostatic

reconstruction for shallow water flows, SIAM J. Sci. Comput., 25 (2004), pp. 2050-2065, https://doi.org/10.1137/S1064827503431090. · Zbl 1133.65308

[16] L. Baiotti, I. Hawke, P. J. Montero, F. Löffler, L. Rezzolla, N. Stergioulas, J. A. Font, and E. Seidel, Three-dimensional relativistic simulations of rotating neutron-star collapse to a Kerr black hole, Phys. Rev. D, 71 (2005), 024035.

[17] F. Banyuls, J. A. Font, J. M. Ibán⊠ez, J. M. Martí, and J. A. Miralles, Numerical 3+1 general relativistic hydrodynamics: A local characteristic approach, Astrophys. J., 476 (1997), pp. 221-231.

[18] C. Bassi, L. Bonaventura, S. Busto, and M. Dumbser, A hyperbolic reformulation of the Serre-Green-Naghdi model for general bottom topographies, Comput. \& Fluids, 212 (2020), 104716. · Zbl 07335348

[19] T. W. Baumgarte and S. L. Shapiro, Numerical integration of Einstein's field equations, Phys. Rev. D, 59 (1998), 024007. · Zbl 1250.83004

[20] T. W. Baumgarte and S. L. Shapiro, Numerical Relativity: Solving Einstein's Equations on the Computer, Cambridge University Press, 2010. · Zbl 1198.83001

[21] J. P. Berberich, P. Chandrashekar, and C. Klingenberg, High order well-balanced finite volume methods for multi-dimensional systems of hyperbolic balance laws, Comput. \& Fluids, 219 (2021), 104858. · Zbl 07426188

[22] A. Bermúdez, X. López, and M. E. Vázquez-Cendón, Numerical solution of non-isothermal non-adiabatic flow of real gases in pipelines, J. Comput. Phys., 323 (2016), pp. 126-148. · Zbl 1415.76463

[23] A. Bermudez and M. E. Vazquez, Upwind methods for hyperbolic conservation laws with source terms, Comput. \& Fluids, 23 (1994), pp. 1049-1071. · Zbl 0816.76052

[24] S. Bernuzzi and D. Hilditch, Constraint violation in free evolution schemes: Comparing the BSSNOK formulation with a conformal decomposition of the Z4 formulation, Phys. Rev. D, 81 (2010), 084003.

[25] C. Bona, T. Ledvinka, C. Palenzuela, and M. Zácek, General-covariant evolution formalism for numerical relativity, Phys. Rev. D, 67 (2003), 104005. · Zbl 1074.83003

[26] C. Bona, T. Ledvinka, C. Palenzuela, and M. Zácek, Symmetry-breaking mechanism for the Z4 general-covariant evolution system, Phys. Rev. D, 69 (2004), 64036. · Zbl 1074.83003

[27] C. Bona, J. Massó, E. Seidel, and J. Stela, New formalism for numerical relativity, Phys. Rev. Lett., 75 (1995), pp. 600-603.

[28] S. Bonazzola, E. Gourgoulhon, P. Grandclement, and J. Novak, Constrained scheme for the Einstein equations based on the Dirac gauge and spherical coordinates, Phys. Rev. D, 70 (2004), 104007.

[29] N. Botta, R. Klein, S. Langenberg, and S. Lützenkirchen, Well balanced finite volume methods for nearly hydrostatic flows, J. Comput. Phys., 196 (2004), pp. 539-565. · Zbl 1109.86304

[30] F. Bouchut, Nonlinear Stability of Finite Volume Methods for Hyperbolic Conservation Laws and Well-Balanced Schemes for Sources, Springer Science + Business Media, 2004. · Zbl 1086.65091

[31] N. Bucciantini and L. Del Zanna, General relativistic magnetohydrodynamics in axisymmetric dynamical spacetimes: The X-ECHO code, Astron. Astrophys., 528 (2011), A101.

[32] N. Bucciantini and L. Del Zanna, A fully covariant mean-field dynamo closure for numerical 3+1 resistive GRMHD, Monthly Not. Roy. Astr. Soc., 428 (2013), pp. 71-85.

[33] M. Bugli, L. Del Zanna, and N. Bucciantini, Dynamo action in thick discs around Kerr black holes: High-order resistive GRMHD simulations, Monthly Not. Roy. Astr. Soc. Lett., 440 (2014), pp. L41-L45.

[34] M. Bugner, Discontinuous Galerkin Methods for General Relativistic Hydrodynamics, Ph.D. thesis, Friedrich-Schiller-Universität Jena, Jena, Germany, 2018.

[35] S. Busto, M. Dumbser, C. Escalante, S. Gavrilyuk, and N. Favrie, On high order ADER discontinuous Galerkin schemes for first order hyperbolic reformulations of nonlinear dispersive systems, J. Sci. Comput., 87 (2021), 48. · Zbl 1465.76060

[36] S. Carroll, Spacetime and Geometry. An Introduction to General Relativity, Addison-Wesley, San Francisco, 2004. · Zbl 1131.83001

[37] M. Castro, J. Gallardo, and C. Parés, High order finite volume schemes based on reconstruction of states for solving hyperbolic systems with nonconservative products. Applications to shallow-water systems, Math. Comp., 75 (2006), pp. 1103-1134. · Zbl 1096.65082

[38] M. Castro, J. M. Gallardo, J. A. López-García, and C. Parés, Well-balanced high order extensions of Godunov's method for semilinear balance laws, SIAM J. Numer. Anal., 46 (2008), pp. 1012-1039, https://doi.org/10.1137/060674879. · Zbl 1159.74045

[39] M. J. Castro, T. M. de Luna, and C. Parés, Well-balanced schemes and path-conservative numerical methods, in Handbook of Numerical Methods for Hyperbolic Problems, Handb. Numer. Anal. 18, Elsevier/North-Holland, Amsterdam, 2017, pp. 131-175. · Zbl 1368.65131

[40] M. J. Castro and C. Parés, Well-balanced high-order finite volume methods for systems of balance laws, J. Sci. Comput., 82 (2020), pp. 1-48. · Zbl 1440.65109

[41] P. Chandrashekar and C. Klingenberg, A second order well-balanced finite volume scheme for Euler equations with gravity, SIAM J. Sci. Comput., 37 (2015), pp. B382-B402, https://doi.org/10.1137/140984373. · Zbl 1320.76078

[42] A. Y. Chernyshenko, M. A. Olshanskii, and Y. V. Vassilevski, A hybrid finite volume-finite element method for bulk-surface coupled problems, J. Comput. Phys., 352 (2018), pp. 516-533. · Zbl 1375.76098

[43] S. Chiocchetti, I. Peshkov, S. Gavrilyuk, and M. Dumbser, High order ADER schemes and GLM curl cleaning for a first order hyperbolic formulation of compressible flow with surface tension, J. Comput. Phys., 426 (2021), 109898.

[44] L. Cirrottola, M. Ricchiuto, A. Froehly, B. Re, A. Guardone, and G. Quaranta, Adaptive deformation of 3D unstructured meshes with curved body fitted boundaries with application to unsteady compressible flows, J. Comput. Phys., 433 (2021), 110177.

[45] I. Cordero-Carrión, P. Cerdá-Durán, H. Dimmelmeier, J. L. Jaramillo, J. Novak, and E. Gourgoulhon, Improved constrained scheme for the Einstein equations: An approach to the uniqueness issue, Phys. Rev. D, 79 (2009), 024017. · Zbl 1222.83024

[46] I. Cordero-Carrión, J. M. Ibanez, E. Gourgoulhon, J. L. Jaramillo, and J. Novak, Mathematical issues in a fully constrained formulation of the Einstein equations, Phys. Rev. D, 77 (2008), 084007.

[47] G. Dal Maso, P. G. Lefloch, and F. Murat, Definition and weak stability of nonconservative products, J. Math. Pures Appl. (9), 74 (1995), pp. 483-548. · Zbl 0853.35068

[48] L. Del Zanna and N. Bucciantini, An efficient shock-capturing central-type scheme for multidimensional relativistic flows. I. Hydrodynamics, Astron. Astrophys., 390 (2002), pp. 1177-1186. · Zbl 1209.76022

[49] L. Del Zanna, O. Zanotti, N. Bucciantini, and P. Londrillo, ECHO: A Eulerian conservative high-order scheme for general relativistic magnetohydrodynamics and magnetodynamics, Astron. Astrophys., 473 (2007), pp. 11-30.

[50] V. Desveaux, M. Zenk, C. Berthon, and C. Klingenberg, A well-balanced scheme to capture non-explicit steady states in the Euler equations with gravity, Internat. J. Numer. Methods Fluids, 81 (2016), pp. 104-127. · Zbl 1382.65310

[51] K. Dionysopoulou, D. Alic, C. Palenzuela, L. Rezzolla, and B. Giacomazzo, General-relativistic resistive magnetohydrodynamics in three dimensions: Formulation and tests, Phys. Rev. D, 88 (2013), 044020.

[52] M. D. Duez, Y. T. Liu, S. L. Shapiro, and B. C. Stephens, Relativistic magnetohydrodynamics in dynamical spacetimes: Numerical methods and tests, Phys. Rev. D, 72 (2005), 024028.

[53] M. Dumbser, S. Chiocchetti, and I. Peshkov, On numerical methods for hyperbolic PDE with curl involutions, in Continuum Mechanics, Applied Mathematics and Scientific Computing: Godunov's Legacy, Springer, 2020, pp. 125-134.

[54] M. Dumbser, F. Fambri, E. Gaburro, and A. Reinarz, On GLM curl cleaning for a first order reduction of the CCZ4 formulation of the Einstein field equations, J. Comput. Phys., 404 (2020), 109088. · Zbl 1453.85002

[55] M. Dumbser, F. Guercilena, S. Köppel, L. Rezzolla, and O. Zanotti, Conformal and covariant Z4 formulation of the Einstein equations: Strongly hyperbolic first-order reduction and solution with discontinuous Galerkin schemes, Phys. Rev. D, 97 (2018), 084053.

[56] M. Dumbser and O. Zanotti, Very high order PNPM schemes on unstructured meshes for the resistive relativistic MHD equations, J. Comput. Phys., 228 (2009), pp. 6991-7006. · Zbl 1261.76028

[57] F. Fambri, M. Dumbser, S. Köppel, L. Rezzolla, and O. Zanotti, ADER discontinuous Galerkin schemes for general-relativistic ideal magnetohydrodynamics, Monthly Not. Roy. Astr. Soc., 477 (2018), pp. 4543-4564.

[58] J. A. Font, Numerical hydrodynamics and magnetohydrodynamics in general relativity, Living Rev. Relativ., 11 (2008), 7. · Zbl 1166.83003

[59] E. Gaburro, A unified framework for the solution of hyperbolic PDE systems using high order direct Arbitrary-Lagrangian-Eulerian schemes on moving unstructured meshes with topology change, Arch. Comput. Methods Eng., 28 (2021), pp. 1249-1321.

[60] E. Gaburro, W. Boscheri, S. Chiocchetti, C. Klingenberg, V. Springel, and M. Dumbser, High order direct Arbitrary-Lagrangian-Eulerian schemes on moving Voronoi meshes with topology changes, J. Comput. Phys., 407 (2020), 109167.

[61] E. Gaburro, M. J. Castro, and M. Dumbser, A well balanced diffuse interface method for complex nonhydrostatic free surface flows, Comput. \& Fluids, 175 (2018), pp. 180-198. · Zbl 1410.76224

[62] E. Gaburro, M. J. Castro, and M. Dumbser, Well-balanced Arbitrary-Lagrangian-Eulerian finite volume schemes on moving nonconforming meshes for the Euler equations of gas dynamics with gravity, Monthly Not. Roy. Astr. Soc., 477 (2018), pp. 2251-2275.

[63] E. Gaburro and M. Dumbser, A posteriori subcell finite volume limiter for general \(P_NP_M\) schemes: Applications from gasdynamics to relativistic magnetohydrodynamics, J. Sci. Comput., 86 (2021), 37. · Zbl 1475.65091

[64] E. Gaburro, M. Dumbser, and M. J. Castro, Direct Arbitrary-Lagrangian-Eulerian finite volume schemes on moving nonconforming unstructured meshes, Comput. \& Fluids, 159 (2017), pp. 254-275. · Zbl 1390.76433

[65] B. Giacomazzo and L. Rezzolla, WhiskyMHD: A new numerical code for general relativistic magnetohydrodynamics, Classical Quantum Gravity, 24 (2007), pp. S235-S258. · Zbl 1117.83002

[66] L. Gosse, A well-balanced scheme using non-conservative products designed for hyperbolic systems of conservation laws with source terms, Math. Models Methods Appl. Sci., 11 (2001), pp. 339-365. · Zbl 1018.65108

[67] L. Grosheintz-Laval and R. Käppeli, High-order well-balanced finite volume schemes for the Euler equations with gravitation, J. Comput. Phys., 378 (2019), pp. 324-343. · Zbl 1416.65266

[68] C. Gundlach and J. M. Martín-García, Hyperbolicity of second order in space systems of evolution equations, Classical Quantum Gravity, 23 (2006), pp. S387-S404. · Zbl 1191.83009

[69] R. Käppeli and S. Mishra, Well-balanced schemes for the Euler equations with gravitation, J. Comput. Phys., 259 (2014), pp. 199-219. · Zbl 1349.76345

[70] R. Käppeli and S. Mishra, A well-balanced finite volume scheme for the Euler equations with gravitation-The exact preservation of hydrostatic equilibrium with arbitrary entropy stratification, Astron. Astrophys., 587 (2016), A94.

[71] K. Kiuchi, Y. Sekiguchi, M. Shibata, and K. Taniguchi, Long-term general relativistic simulation of binary neutron stars collapsing to a black hole, Phys. Rev. D, 80 (2009), 064037.

[72] C. Klingenberg, G. Puppo, and M. Semplice, Arbitrary order finite volume well-balanced schemes for the Euler equations

with gravity, SIAM J. Sci. Comput., 41 (2019), pp. A695-A721, https://doi.org/10.1137/18M1196704. · Zbl 1412.65125

[73] S. Komissarov, General relativistic magnetohydrodynamic simulations of monopole magnetospheres of black holes, Monthly Not. Roy. Astr. Soc., 350 (2004), pp. 1431-1436.

[74] R. J. LeVeque, Balancing source terms and flux gradients in high-resolution Godunov methods: The quasi-steady wave-propagation algorithm, J. Comput. Phys., 146 (1998), pp. 346-365. · Zbl 0931.76059

[75] J. M. Martí and E. Müller, Grid-based methods in relativistic hydrodynamics and magnetohydrodynamics, Living Rev. Comput. Astrophys., 1 (2015), 3.

[76] F. C. Michel, Accretion of matter by condensed objects, Astrophys. Space Sci., 15 (1972), pp. 153-160.

[77] T. Nakamura, K. Oohara, and Y. Kojima, General relativistic collapse to black holes and gravitational waves from black holes, Progr. Theoret. Phys. Suppl., no. 90 (1987), pp. 1-218.

[78] J. R. Oppenheimer and G. M. Volkoff, On massive neutron cores, Phys. Rev., 55 (1939), pp. 374-381. · Zbl 0020.28501

[79] C. Palenzuela, L. Lehner, O. Reula, and L. Rezzolla, Beyond ideal MHD: Towards a more realistic modelling of relativistic astrophysical plasmas, Monthly Not. Roy. Astr. Soc., 394 (2009), pp. 1727-1740.

[80] C. Parés, Numerical methods for nonconservative hyperbolic systems: A theoretical framework, SIAM J. Numer. Anal., 44 (2006), pp. 300-321, https://doi.org/10.1137/050628052. · Zbl 1130.65089

[81] B. Perthame and C. Simeoni, A kinetic scheme for the Saint-Venant system with a source term, Calcolo, 38 (2001), pp. 201-231. · Zbl 1008.65066

[82] O. Porth, H. Olivares, Y. Mizuno, Z. Younsi, L. Rezzolla, M. Moscibrodzka, H. Falcke, and M. Kramer, The black hole accretion code, Comput. Astrophys. Cosmol., 4 (2017), pp. 1-42.

[83] D. Radice and L. Rezzolla, THC: A new high-order finite-difference high-resolution shock-capturing code for special-relativistic hydrodynamics, Astron. Astrophys., 547 (2012), A26.

[84] D. Radice, L. Rezzolla, and F. Galeazzi, Beyond second-order convergence in simulations of binary neutron stars in full general relativity, Monthly Not. Roy. Astr. Soc. Lett., 437 (2013), pp. L46-L50.

[85] T. C. Rebollo, A. D. Delgado, and E. D. F. Nieto, A family of stable numerical solvers for the shallow water equations with source terms, Comput. Methods Appl. Mech. Engrg., 192 (2003), pp. 203-225. · Zbl 1083.76557

[86] L. Rezzolla and O. Zanotti, Relativistic Hydrodynamics, Oxford University Press, 2013.

[87] G. Russo and A. Anile, Stability properties of relativistic shock waves: Basic results, Phys. Fluids, 30 (1987), pp. 2406-2413. · Zbl 0663.76143

[88] K. Schaal, A. Bauer, P. Chandrashekar, R. Pakmor, C. Klingenberg, and V. Springel, Astrophysical hydrodynamics with a high-order discontinuous Galerkin scheme and adaptive mesh refinement, Monthly Not. Roy. Astr. Soc., 453 (2015), pp. 4278-4300.

[89] M. Shibata and T. Nakamura, Evolution of three-dimensional gravitational waves: Harmonic slicing case, Phys. Rev. D, 52 (1995), pp. 5428-5444. · Zbl 1250.83027

[90] V. Springel, E pur si muove: Galilean-invariant cosmological hydrodynamical simulations on a moving mesh, Monthly Not. Roy. Astr. Soc., 401 (2010), pp. 791-851.

[91] R. Takahashi and M. Umemura, General relativistic radiative transfer code in rotating black hole space-time: ARTIST, Monthly Not. Roy. Astr. Soc., 464 (2016), pp. 4567-4585.

[92] H. Tang, T. Tang, and K. Xu, A gas-kinetic scheme for shallow-water equations with source terms, Z. Angew. Math. Phys., 55 (2004), pp. 365-382. · Zbl 1142.76457

[93] A. Thomann, M. Zenk, and C. Klingenberg, A second-order positivity-preserving well-balanced finite volume scheme for Euler equations with gravity for arbitrary hydrostatic equilibria, Internat. J. Numer. Methods Fluids, 89 (2019), pp. 465-482.

[94] R. C. Tolman, Static solutions of Einstein's field equations for spheres of fluid, Phys. Rev., 55 (1939), pp. 364-373. · Zbl 65.1048.02

[95] E. F. Toro, Riemann Solvers and Numerical Methods for Fluid Dynamics, 2nd ed., Springer, 1999. · Zbl 0923.76004

[96] B. Van Leer, Towards the ultimate conservative difference scheme. II. Monotonicity and conservation combined in a second-order scheme, J. Comput. Phys., 14 (1974), pp. 361-370. · Zbl 0276.65055

[97] B. Van Leer, Towards the ultimate conservative difference scheme. V. A second-order sequel to Godunov's method, J. Comput. Phys., 32 (1979), pp. 101-136. · Zbl 1364.65223

[98] R. M. Wald, General Relativity, University of Chicago Press, Chicago, 1984. · Zbl 0549.53001

[99] C. J. White, J. M. Stone, and C. F. Gammie, An extension of the Athena++ code framework for GRMHD based on advanced Riemann solvers and staggered-mesh constrained transport, Astrophys. J. Suppl. Ser., 225 (2016), 22.

[100] J. R. Wilson, Some Magnetic Effects in Stellar Collapse and Accretion, Tech. report, Lawrence Livermore Laboratory, University of California, Livermore, CA, 1975.

[101] Y. Xing, Exactly well-balanced discontinuous Galerkin methods for the shallow water equations with moving water equilibrium, J. Comput. Phys., 257 (2014), pp. 536-553. · Zbl 1349.76289

[102] Y. Xing and C.-W. Shu, High order well-balanced finite volume WENO schemes and discontinuous Galerkin methods for a class of hyperbolic systems with source terms, J. Comput. Phys., 214 (2006), pp. 567-598. · Zbl 1089.65091

paper as accurately as possible without claiming the completeness or perfect precision of the matching.