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**Efficient formulation of the Gibbs-Appell equations for constrained multibody systems.**

(English) [Zbl 1483.70025](#)

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**Summary:** In this study, we present explicit equations of motion for general mechanical systems exposed to holonomic and nonholonomic constraints based on the Gibbs-Appell formulation. Without constructing the Gibbs function, the proposed method provides a minimal set of first-order dynamic equations applicable for multibody constrained systems. Transforming the Newton-Euler equations from physical coordinates to quasivelocities spaces eliminate constraint reaction forces from motion equations. In this study, we develop a general procedure to select effective quasivelocities, which indicate that the proposed quasivelocities satisfy constraints, eliminate Lagrange multipliers, and reduce the number of dynamic equations to degrees of freedom. Besides, we test the validity and efficiency of the proposed approach using three constrained dynamical systems as illustrative examples. Finally, we compare the simulation results with Udwadia-Kalaba, augmented Lagrangian, and other conventional methods.

**MSC:**

**70E55** Dynamics of multibody systems

**Keywords:**

explicit equations of motion; Gibbs-Appell equations; Newton-Euler equations; nonholonomic constraint equations; quasivelocities; elimination of Lagrange multipliers

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