Clark, Ed; **Katzourakis, Nikos**; **Muha, Boris**

Vectorial variational problems in L^∞ constrained by the Navier-Stokes equations. (English)

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MSC:

35Q30 Navier-Stokes equations
35D35 Strong solutions to PDEs
35A15 Variational methods applied to PDEs
76D05 Navier-Stokes equations for incompressible viscous fluids
49J40 Variational inequalities
49K20 Optimality conditions for problems involving partial differential equations
49K35 Optimality conditions for minimax problems
49M41 PDE constrained optimization (numerical aspects)

Keywords:

Navier-Stokes equations; calculus of variations in L^∞; PDE-constrained optimisation; Euler-Lagrange equations; Aronsson-Euler systems; data assimilation

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