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**Three-level order-adaptive weighted essentially non-oscillatory schemes.** (English)

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Summary: Classical fifth-order weighted essentially non-oscillatory (WENO) schemes are based on reconstructions from three consecutive second-order polynomials. They give a third-order accurate scheme when two consecutive polynomials are smooth and one is non-smooth. This situation means losing one order of accuracy if compared with a fourth-order scheme. In classical WENO schemes, weighting functions that determine the weights for the polynomials and that eventually define the reconstruction are directly related to a smoothness indicator. This property may lead to a non-zero residual of a very small value in the Taylor expansion. To handle this situation a weighting procedure that does not directly rely on the smoothness indicator is presented. Once the smooth polynomials are determined, the smoothness indicator has no role in the further steps, which are taken care of by a new switch function. Contrary to other approaches this switch function does not involve any conditional statements. A tuning parameter is also included so that the resulting order-adaptive property can be adjusted to specific requirements. The performance of the resulting schemes for cell-average and point-value reconstructions is studied. In addition, the effect of the choice of numerical flux or Riemann solver on these reconstruction strategies is analysed. Numerical tests for the Euler equations of gas dynamics in one and two space dimensions are presented. Improvements in resolution and the stability are found when compared with the conventional WENO techniques.

**MSC:**

[65-XX](#) Numerical analysis

[35F61](#) Initial-boundary value problems for systems of nonlinear first-order PDEs

[35L50](#) Initial-boundary value problems for first-order hyperbolic systems

[35Q31](#) Euler equations

[76N15](#) Gas dynamics (general theory)

[65M06](#) Finite difference methods for initial value and initial-boundary value problems involving PDEs

[65M08](#) Finite volume methods for initial value and initial-boundary value problems involving PDEs

[65M12](#) Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs

[76J20](#) Supersonic flows

[76Q05](#) Hydro- and aero-acoustics

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**Keywords:**

WENO; ENO; high resolution schemes; order-adaptive schemes; finite volume method

**Software:**

HE-E1GODF

**Full Text:** [DOI](#)

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