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Theoretical development of continuum dislocation dynamics for finite-deformation crystal plasticity at the mesoscale. (English) [Zbl 1477.74088](#)

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Summary: The equations of dislocation transport at finite crystal deformation were developed, with a special emphasis on a vector density representation of dislocations. A companion thermodynamic analysis yielded a generalized expression for the driving force of dislocations that depend on Mandel (Cauchy) stress in the reference (spatial) configurations and the contribution of the dislocation core energy to the free energy of the crystal. Our formulation relied on several dislocation density tensor measures linked to the incompatibility of the plastic distortion in the crystal. While previous works develop such tensors starting from the multiplicative decomposition of the deformation gradient, we developed the tensor measures of the dislocation density and the dislocation flux from the additive decomposition of the displacement gradient and the crystal velocity fields. The two-point dislocation density measures defined by the referential curl of the plastic distortion and the spatial curl of the inverse elastic distortion and the associate dislocation currents were found to be more useful in deriving the referential and spatial forms of the transport equations for the vector density of dislocations. A few test problems showing the effect of finite deformation on the static dislocation fields are presented, with a particular attention to lattice rotation. The framework developed provides the theoretical basis for investigating crystal plasticity and dislocation patterning at the mesoscale, and it bears the potential for realistic comparison with experiments upon numerical solution.

MSC:

[74N05](#) Crystals in solids

[74A45](#) Theories of fracture and damage

[74C99](#) Plastic materials, materials of stress-rate and internal-variable type

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