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Proportional integral regulation control of a one-dimensional semilinear wave equation.

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Authors' abstract: This paper is concerned with the proportional integral (PI) regulation control of the left Neumann trace of a one-dimensional semilinear wave equation. The control input is selected as the right Neumann trace. The control design goes as follows. First, a preliminary (classical) velocity feedback is applied in order to shift all but a finite number of the eivenvalues of the underlying unbounded operator into the open left half-plane. We then leverage the projection of the system trajectories into an adequate Riesz basis to obtain a truncated model of the system capturing the remaining unstable modes. The controller is computed by applying a classical PI control design scheme to this truncated model. Local stability of the resulting closed-loop infinite-dimensional system is obtained through the study of an adequate Lyapunov function. Finally, an estimate assessing the set point tracking performance of the left Neumann trace is derived.

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MSC:

35L20 Initial-boundary value problems for second-order hyperbolic equations
35L71 Second-order semilinear hyperbolic equations
93C20 Control/observation systems governed by partial differential equations

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References:

[1] Z. Artstein, Linear systems with delayed controls: A reduction, IEEE Trans. Automat. Control, 27 (1982), pp. 869-879. · Zbl 0486.93011

[2] K. J. \AAström and T. Hägglund, PID Controllers: Theory, Design, and Tuning, 2nd ed., Instrument Society of America, Research Triangle Park, NC, 1995.

[3] K. J. \AAström and R. M. Murray, Feedback Systems: An Introduction for Scientists and Engineers, Princeton University Press, Princeton, NJ, 2021.

[4] M. Barreau, F. Gouaisbaut, and A. Seuret, Practical stability analysis of a drilling pipe under friction with a PI-controller, IEEE Trans. Control Syst. Technol., 29 (2021), pp. 620-634.

[5] G. Bastin and J.-M. Coron, Exponential stability of PI control for Saint-Venant equations with a friction term, Methods Appl. Anal., 26 (2019), pp. 101-112. · Zbl 1436.35035

[6] G. Bastin, J.-M. Coron, and S. O. Tamasoiu, Stability of linear density-flow hyperbolic systems under PI boundary control, Automatica, 53 (2015), pp. 37-42. · Zbl 1371.93160

[7] D. Bresch-Pietri, C. Prieur, and E. Trélat, New formulation of predictors for finite-dimensional linear control systems with input delay, Systems Control Lett., 113 (2018), pp. 9-16. · Zbl 1386.93127

[8] J.-M. Coron and A. Hayat, PI controllers for 1-D nonlinear transport equation, IEEE Trans. Automat. Control, 64 (2019), pp. 4570-4582. · Zbl 1482.93270

[9] J.-M. Coron and E. Trélat, Global steady-state controllability of one-dimensional semilinear heat equations, SIAM J. Control Optim., 43 (2004), pp. 549-569. · Zbl 1101.93011

[10] J.-M. Coron and E. Trélat, Global steady-state stabilization and controllability of 1D semilinear wave equations, Commun. Contemp. Math., 8 (2006), pp. 535-567. · Zbl 1101.93039

[11] R. Datko, Two examples of ill-posedness with respect to time delays revisited, IEEE Trans. Automat. Control, 42 (1997), pp. 511-515. · Zbl 0878.73046

[12] V. Dos Santos, G. Bastin, J.-M. Coron, and B. d'Andréa Novel, Boundary control with integral action for hyperbolic systems of conservation laws: Stability and experiments, Automatica, 44 (2008), pp. 1310-1318. · Zbl 1283.93211

[13] V. Dos Santos Martins, Y. Wu, and M. Rodrigues, Design of a proportional integral control using operator theory for infinite dimensional hyperbolic systems, IEEE Trans. Control Syst. Technol., 22 (2014), pp. 2024-2030.

[14] I. Gohberg and M. G. Krein, Introduction to the Theory of Linear Nonselfadjoint Operators, vol. 18, American Mathematical Society, Providence, RI, 1978. · Zbl 0181.13504

[15] B.-Z. Guo, Riesz basis approach to the stabilization of a flexible beam with a tip mass, SIAM J. Control Optim., 39 (2001), pp. 1736-1747. · Zbl 1183.93111

[16] A. Hayat, PI Controller for the General Saint-Venant Equations, preprint, hal-01827988, 2019.

[17] P.-O. Lamare and N. Bekiaris-Liberis, Control of 2 x 2 linear hyperbolic systems: Backstepping-based trajectory generation and PI-based tracking, Systems Control Lett., 86 (2015), pp. 24-33. · Zbl 1325.93049

[18] H. Lhachemi and C. Prieur, Finite-dimensional observer-based PI regulation control of a reaction-diffusion equation, IEEE Trans. Automat. Control, (2021), https://doi.org/10.1109/TAC.2021.3130874. · Zbl 07352210

[19] H. Lhachemi and C. Prieur, Feedback stabilization of a class of diagonal infinite-dimensional systems with delay boundary control, IEEE Trans. Automat. Control, 66 (2021), pp. 105-120. · Zbl 07320135

[20] H. Lhachemi and C. Prieur, Finite-dimensional observer-based boundary stabilization of reaction-diffusion equations with either a Dirichlet or Neumann boundary measurement, Automatica, 135 (2022), 109955. · Zbl 1480.93199

[21] H. Lhachemi, C. Prieur, and E. Trélat, PI regulation of a reaction-diffusion equation with delayed boundary control, IEEE Trans. Automat. Control, 66 (2021), pp. 1573-1587. · Zbl 07352210

[22] H. Lhachemi, R. Shorten, and C. Prieur, Exponential input-to-state stabilization of a class of diagonal boundary control systems with delay boundary control, Systems Control Lett., 138 (2020), 104651. · Zbl 1436.93123

[23] H. Logemann and H. Zwart, On robust PI-control of infinite-dimensional systems, SIAM J. Control Optim., 30 (1992), pp. 573-593. · Zbl 0756.93036

[24] A. Pazy, Semigroups of linear operators and applications to partial differential equations, vol. 44, Springer, New York, 2012. · Zbl 0516.47023

[25] S. Pohjolainen, Robust multivariable PI-controller for infinite dimensional systems, IEEE Trans. Automat. Control, 27 (1982), pp. 17-30. · Zbl 0493.93029

[26] S. Pohjolainen, Robust controller for systems with exponentially stable strongly continuous semigroups, J. Math. Anal. Appl., 111 (1985), pp. 622-636. · Zbl 0577.93037

[27] M. Rodrigues, Y. Wu, S. Aberkane, and V. D. S. Martins, LMI \& BMI technics for the design of a PI control for irrigation channels, in 2013 European Control Conference (ECC), IEEE, 2013, pp. 3895-3900.

[28] D. L. Russell, Controllability and stabilizability theory for linear partial differential equations: Recent progress and open questions, SIAM Rev., 20 (1978), pp. 639-739. · Zbl 0397.93001

[29] M. Schmidt and E. Trélat, Controllability of Couette flows, Commun. Pure Appl. Anal., 5 (2006), pp. 201-211. · Zbl 1129.76019

[30] A. Terrand-Jeanne, V. Andrieu, V. D. S. Martins, and C.-Z. Xu, Lyapunov functionals for output regulation of exponentially stable semigroups via integral action and application to hyperbolic systems, in 2018 IEEE Conference on Decision and Control (CDC), Miami Beach, FL, 2018, pp. 4631-4636.

[31] A. Terrand-Jeanne, V. Andrieu, V. D. S. Martins, and C.-Z. Xu, Adding Integral Action for Open-Loop Exponentially Stable Semigroups and Application to Boundary Control of PDE Systems, preprint, arXiv:1901.02208, 2019, https://arxiv.org/abs/1901.02208. · Zbl 07320031

[32] A. Terrand-Jeanne, V. Andrieu, M. Tayakout-Fayolle, and V. D. S. Martins, Regulation of inhomogeneous drilling model with a PI controller, IEEE Trans. Automat. Control, 65 (2020), pp. 58-71. · Zbl 1483.93456

[33] N.-T. Trinh, V. Andrieu, and C.-Z. Xu, Design of integral controllers for nonlinear systems governed by scalar hyperbolic partial differential equations, IEEE Trans. Automat. Control, 62 (2017), pp. 4527-4536. · Zbl 1390.93404

[34] M. Tucsnak and G. Weiss, Observation and Control for Operator Semigroups, Springer Science \& Business Media, New York, 2009. · Zbl 1188.93002

[35] C.-Z. Xu and H. Jerbi, A robust PI-controller for infinite-dimensional systems, Int. J. Control, 61 (1995), pp. 33-45. · Zbl 0820.93036

[36] C.-Z. Xu and G. Sallet, Multivariable boundary PI control and regulation of a fluid flow system, Math. Control Relat. Fields, 4 (2014), pp. 501-520. · Zbl 1303.93095