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**Vanishing viscosity limit of the compressible Navier-Stokes equations with finite energy and total mass.** (English) Zbl 1481.35330

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Summary: Assume the initial data of compressible Euler equations has finite energy and total mass. We can construct a sequence of solutions of one-dimensional compressible Navier-Stokes equations (density-dependent viscosity) with stress-free boundary conditions, so that, up to a subsequence, the sequence of solutions of compressible Navier-Stokes equations converges to a finite-energy weak solution of compressible Euler equations. Hence the inviscid limit of the compressible Navier-Stokes is justified. It is worth pointing out that our result covers the interesting case of the Saint-Venant model for shallow water (i.e., $\alpha = 1$, $\gamma = 2$).

**MSC:**

| | |
|---|---|
| 35Q35 | PDEs in connection with fluid mechanics |
| 35Q31 | Euler equations |
| 35B25 | Singular perturbations in context of PDEs |
| 35B44 | Blow-up in context of PDEs |
| 35L65 | Hyperbolic conservation laws |
| 35L67 | Shocks and singularities for hyperbolic equations |
| 76N10 | Existence, uniqueness, and regularity theory for compressible fluids and gas dynamics |
| 35R09 | Integro-partial differential equations |
| 35R35 | Free boundary problems for PDEs |
| 35D30 | Weak solutions to PDEs |
| 76X05 | Ionized gas flow in electromagnetic fields; plasmic flow |
| 76B15 | Water waves, gravity waves; dispersion and scattering, nonlinear interaction |
| 76N17 | Viscous-inviscid interaction for compressible fluids and gas dynamics |

**Keywords:**

Euler equations; Navier-Stokes equations; vanishing viscosity; compensated compactness framework; free boundary; density-dependent viscosity

**Full Text:** DOI

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