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A generalised Serre-Green-Naghdi equations for variable rectangular open channel hydraulics and its finite volume approximation. (English) [Zbl 1481.76028](#)

Muñoz-Ruiz, María Luz (ed.) et al., Recent advances in numerical methods for hyperbolic PDE systems. NumHyp 2019. Selected papers based on the presentations at the 6th international conference on numerical methods for hyperbolic problems, Málaga, Spain, June 17–21, 2019. Cham: Springer. SEMA SIMAI Springer Ser. 28, 251-268 (2021).

Summary: We present a non-linear dispersive shallow water model which enters in the framework of section-averaged models. These new equations are derived up to the second order of the shallow water approximation starting from the three-dimensional incompressible and irrotational Euler system. The derivation is carried out in the case of non-uniform rectangular section and it generalises the well-known one-dimensional Serre-Green-Naghdi (SGN) equations on uneven bottom. The section-averaged model is asymptotically consistent with the Euler system in terms of mass, momentum, and energy equation which provides the richness of content for this model. We propose a well-balanced finite volume approximation and we present some numerical results to show the influence of the section variation.

For the entire collection see [\[Zbl 1470.65004\]](#).

MSC:

76B07 Free-surface potential flows for incompressible inviscid fluids

76M12 Finite volume methods applied to problems in fluid mechanics

Keywords:

Euler equations; second-order asymptotic approximation; free surface shallow water equations; non-hydrostatic pressure; section-averaged dispersive model; finite volume method

Full Text: [DOI](#)

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