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Lukáčová-Medviďová, Mária; Mizerová, Hana; She, Bangwei

New invariant domain preserving finite volume schemes for compressible flows. (English)

Zbl 1481.76146

Muñoz-Ruiz, María Luz (ed.) et al., Recent advances in numerical methods for hyperbolic PDE systems. NumHyp 2019. Selected papers based on the presentations at the 6th international conference on numerical methods for hyperbolic problems, Málaga, Spain, June 17–21, 2019. Cham: Springer. SEMA SIMAI Springer Ser. 28, 131-153 (2021).

Summary: We present new invariant domain preserving finite volume schemes for the compressible Euler and Navier-Stokes-Fourier systems. The schemes are entropy stable and preserve positivity of density and internal energy. More importantly, their convergence towards a strong solution of the limit system has been proved rigorously in [*E. Feireisl* et al., Numer. Math. 144, No. 1, 89–132 (2020; Zbl 1435.65137); " On the convergence of a finite volume method for the Navier-Stokes-Fourier system", IMA J. Numer. Anal. 41, No. 4, 2388-2422 (2021; doi:10.1093/imanum/draa060)]. We will demonstrate their accuracy and robustness on a series of numerical experiments.

For the entire collection see [Zbl 1470.65004].

MSC:

76M12 Finite volume methods applied to problems in fluid mechanics
76N15 Gas dynamics (general theory)
76N06 Compressible Navier-Stokes equations
65M12 Stability and convergence of numerical methods for initial value and initial-boundary value problems involving PDEs

Keywords:

compressible Euler equtions; compressible Navier-Stokes-Fourier equations; finite volume method; invariant domain preserving property; entropy stability; convergence

Full Text: DOI

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